Interpreting Unary, Binary, and Ternary Mixture Permeation Across a SAPO-34 Membrane with Loading-Dependent Maxwell-**Stefan Diffusivities**

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Unary permeation experiments were performed with He, H_2 , CO_2 , N_2 , O_2 , CO , Ar , and CH_4 across a SAPO-34 membrane at 295 K with upstream pressures ranging to 7.2 MPa, ensuring that high occupancies within the zeolite layer are reached. Careful analysis of the unary permeation experiments shows that the Maxwell-Stefan (M-S) diffusivity \hat{P}_i is loading-dependent for all gases except He and H₂. The model of Reed and Ehrlich (*Surf. Sci.* **1981**, 102 , 588-609) was used to describe the loading-dependence of \hat{D}_i . The zero-loading diffusivity $\hat{P}_i(0)$ and the Reed-Ehrlich parameter ϕ_i are backed out from the permeation experiments, providing a complete description of $\hat{\mathbf{D}}_i$ as a function of loading, q_i . Permeation experiments for CO₂/CH₄, CO₂/N₂, N_2/CH_4 , H_2/CH_4 , H_2/N_2 , H_2/CO , and $CO_2/CH_4/N_2$ mixtures were performed to determine to what extent loadingdependent M-S diffusivities influence permeation fluxes and selectivities. Good agreement between the M-S mixture model and experiments are obtained, provided the binary exchange coefficients \hat{D}_{ii} in the M-S uations are assumed to have high enough values to ensure that intercage hops of the molecules occur practically independently of one other.

1. Introduction

A proper description of diffusion of molecules within nanoporous materials such as zeolites, carbon nanotubes, and metal-organic frameworks is required in many potential separation and reaction applications.¹⁻⁵ The Maxwell-Stefan formulation for single component diffusion in zeolites is commonly expressed as

$$
N_i = -\rho \mathbf{D}_i \frac{q_i}{RT} \frac{\mathrm{d}\mu_i}{\mathrm{d}x} \text{ (pure component } i)
$$
 (1)

where N_i is the molar flux expressed in mol m⁻² s⁻¹, ρ is the zeolite framework density expressed in kg m^{-3} , q_i is the molar loading expressed in mol kg⁻¹, $d\mu$ ^{*i*} */dx* is the chemical potential gradient, *R* is the gas constant, *T* is the absolute temperature in K, and \overline{D}_i is the Maxwell-Stefan (M-S) diffusivity, or "corrected" diffusivity of species *i*. In several classic publications on the subject of diffusion in zeolites, it has been customary to assume that $\hat{\mathbf{P}}$ is independent of the loading q_i within the zeolite.6 However, in recent years there has been increasing evidence, both from experiments⁷⁻¹⁰ and from molecular dynamics (MD) simulations¹¹⁻¹⁶ that \overline{D}_i is generally a strong function of loading. For any given molecule, say CH₄, the $\overline{D_i}$ *qi* relation appears to be dependent both on the zeolite topology and connectivity.^{11,17-20} For zeolites structures consisting of cages separated by narrow windows, such as LTA, CHA, ERI, and DDR, the \hat{D}_i increases sharply with loading q_i , reaches a maximum, and eventually decreases to near-zero values as the saturation loading $q_{i, \text{sat}}$ is approached.^{13,17,18,21} The loadingdependence in such structures has been interpreted in terms of a reduction in the free energy barrier for intercage hopping with increased intracage occupancy.18 The approach of Reed and $Ehrlich²² has been applied to cage-type zedite structures to yield$ a simple analytical model to describe the dependence of \hat{P}_i on fractional occupancy θ_i , defined as

$$
\theta_i = \frac{q_i}{q_{i, \text{sat}}} \tag{2}
$$

The primary objective of this paper is to investigate the loadingdependence of $\hat{\mathbf{p}}_i$ in a (Si_{0.061}Al_{0.483}P_{0.455})O₂ (SAPO-34) (an isotype of CHA) membrane by carrying out unary permeation experiments with a variety of light gases (He, H_2 , CO₂, N₂, O₂, CO, Ar, and CH4). For this purpose, experiments in an improved membrane were performed with upstream pressures ranging to 7.2 MPa to ensure high occupancies in the membrane. The membrane synthesis procedure used in the present study used a seeding procedure that resulted in a thinner membrane and higher fluxes than obtained in a previous study.²³ The details of membrane synthesis, characterization, measurement procedures along with the experimental data have been given in the Supporting Information. The current work supplements the unary permeation data for CO_2 , N_2 , and CH_4 reported in our earlier publication.24

In our earlier publication, 24 we had analyzed experimental data on permeation of binary mixtures $CO₂/CH₄, CO₂/N₂$, and $CH₄/N₂$ and concluded that for modeling purposes, the binary exchange coefficient \mathbf{D}_{12} in the M-S formulation is best taken to have a large enough value to ensure that intercage jumps of molecules occur independently of one another. In the current work, we report additional binary permeation data for H_2/CH_4 , $H₂/N₂$, and $H₂/CO$ mixtures to examine whether the conclusion reached in the earlier paper on independent intercage hopping

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Figure 1. Pure component isotherm data for pure (a) CO_2 , CH_4 , N_2 , (b) CO, Ar, O_2 , H₂, and (c) He and along with fits using eq 3 or eq 5, denoted by the continuous solid lines. The He isotherm data is from GCMC simulations.

of molecules holds also for mixtures containing H₂. Furthermore, we report data for ternary $CO_2/CH_4/N_2$ mixture permeation experiments and examine whether the M-S formulation with infinite binary exchange coefficients \mathbf{P}_{ij} for each of the binary pairs is successful in predicting the permeation fluxes in the ternary mixture.

2. Fitting of Pure Component Isotherms

Adsorption experiments were carried out in an Autosorb-1 (Quantachrome Corp. Model AS1-C-VP-RGA) system. The experimental pure component isotherms for pure $CO₂$, $CH₄$, N₂,

TABLE 1: Pure Component Isotherm Fit Data Using Eq 3*^a*

molecule	b;	Ω,	$q_{i,\text{sat}}$
CO ₂	7.67×10^{-5}	6	8.2
CH ₄	5.87×10^{-6}	6	8.2
N_2	1.26×10^{-6}	6	8.2
H ₂	2.84×10^{-7}		12.3
O ₂	1.2×10^{-6}	6	8.2
CO	2.31×10^{-6}	6	8.2
Ar	1.26×10^{-6}		82

^a b_i is expressed in Pa⁻¹, Ω_i is in molecules per cage, and $q_{i, \text{sat}}$ is in mol kg^{-1} .

CO, Ar, O_2 , and H₂ at 295 K are plotted in Figure 1a-c. For adsorption in zeolite structures with cages separated by narrow windows, such as SAPO-34, the model based on statistical thermodynamics described in Chapter 3 of Ruthven²⁵ is particularly relevant and useful.

$$
b_{i}f_{i} + \sum_{m=2}^{\Omega_{i}} \frac{(b_{i}f_{i})^{m}}{(m-1)!} \left[\frac{1 - \frac{m}{\Omega_{i} + 1}}{1 - \frac{1}{\Omega_{i} + 1}} \right]^{m}
$$

$$
q_{i} = \frac{q_{i, \text{sat}}}{\Omega_{i}}
$$

$$
1 + b_{i}f_{i} + \sum_{m=2}^{\Omega_{i}} \frac{(b_{i}f_{i})^{m}}{(m)!} \left[\frac{1 - \frac{m}{\Omega_{i} + 1}}{1 - \frac{1}{\Omega_{i} + 1}} \right]^{m}
$$
(3)

In eq 3, q_i represents the loading in mol kg^{-1} , $q_{i, sat}$ is the saturation loading, and Ω_i is maximum capacity expressed in molecules per cage. On the basis of the atomic composition of SAPO-34 used in our experiments, $(Si_{0.061}Al_{0.483}P_{0.455})O_2$, we calculate

$$
q_{i, \text{sat}} = 1.369 \Omega_i \tag{4}
$$

It is important to have a good estimate of the saturation capacity, *qi*,sat, but it is not possible to obtain this information from experimental isotherm data because the pressures at which the pores get saturated can range to about 10^9 Pa; our experimental isotherms were only determined to a maximum pressure of 110 kPa. For $CO₂$ for example, the molar volume of the liquid phase at 109 Pa is estimated to lie in the range of $30-35$ cm³/mol from the data given by Mäder and Berman.²⁶ Golden and Sircar²⁷ estimate the molar volumes for H_2 , CO_2 , N₂, Ar, and CH₄ to be 22.6, 33.3, 37.7, 26.3, and 31.6 cm³/ mol, respectively. Of these gases, $CO₂$ has the highest adsorption strength and the loading of $CO₂$ at the highest pressures used in the feed in the permeation experiments, to be discussed below, will be close to its saturation value. For the measured pore volume of $0.26 \text{ cm}^3/\text{g}$, the maximum capacity after rounding off to integer values is estimated to be 6 molecules per cage for CO_2 , corresponding to 8.2 mol kg⁻¹. For H₂, using the value of 22.6 cm3/mol for the molar volume results in a saturation capacity after rounding off of $\Omega_i = 9$ molecules per cage. Table 1 summarizes the values of the fit parameter b_i in eq 3.

For He, the measurement of the adsorption isotherm using the Autosorb-1 experimental setup was not feasible. In this case, we used Grand Canonical Monte Carlo (GCMC) simulations using the force field described in the literature²⁸ to determine the Henry coefficient H_i defined by

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$$
q_i = H_i f_i \quad \text{(Henry regime)} \tag{5}
$$

The GCMC simulation data for all-silica CHA are presented in Figure 1c, and the best-fit value is $H_i = 1.37 \times 10^{-7}$ mol kg^{-1} Pa⁻¹; this value is assumed to be valid also for SAPO-34.

3. Analysis of Unary Permeation Data

If the M-S diffusivity $\hat{\mathbf{D}}_i$ can be taken to be loadingindependent, we may write the permeation flux as

$$
N_i = \frac{\rho \mathbf{D}_i}{\delta} DF_i \tag{6}
$$

where DF_i is the driving force

$$
DF_i \equiv \int_{f_{i,\text{up}}}^{f_{i,\text{down}}} \frac{q_i}{f_i} df_i \tag{7}
$$

where $f_{i,\text{up}}$ and $f_{i,\text{down}}$ represent the pressures (more strictly, fugacities) of component *i* at the upstream (retentate) and downstream (permeate) sides of the membrane. For He permeation, with eq 5 describing the adsorption isotherm in the Henry regime, the DF_i can be calculated from

$$
DF_i = H_i \left(f_{i, \text{up}} - f_{i, \text{down}} \right) \tag{8}
$$

For all other gases, the adsorption isotherms are described by eq 3 and the integral in eq 7 can be evaluated analytically to obtain

$$
DF_{i} = \frac{q_{i, \text{sat}}}{\Omega_{i}} \ln \left(\frac{1 + b_{i} f_{i, \text{up}} + \sum_{m=2}^{\Omega_{i}} \frac{(b_{i} f_{i, \text{up}})^{m}}{(m)!} \left(\frac{1 - \frac{m}{\Omega_{i} + 1}}{1 - \frac{1}{\Omega_{i} + 1}} \right)^{m}}{1 + b_{i} f_{i, \text{down}} + \sum_{m=2}^{\Omega_{i}} \frac{(b_{i} f_{i, \text{down}})^{m}}{(m)!} \left(\frac{1 - \frac{m}{\Omega_{i} + 1}}{1 - \frac{1}{\Omega_{i} + 1}} \right)^{m} \right) \right)
$$
(9)

Plots of the unary permeation fluxes for He and H_2 against the driving force DF_i are shown in Figure 2. The linear dependence of the flux N_i on DF_i confirms that the assumption of a loading-independent \mathbf{D}_i is a good one for He and H₂. The straight lines have been drawn taking the fitted value of $\rho \mathbf{D}_i / \delta$ $= 0.322$ and 0.293 kg m⁻² s⁻¹ for He and H₂, respectively.

The plots of N_i vs DF_i for CO_2 , N_2 , O_2 , CO , Ar, and CH_4 are shown in Figure 3a,b. For none of these gases is the N_i - DF_i relationship precisely linear. This is more clearly evident in Figure 4a,b in which the values of the transport coefficients, $\rho\mathbf{D}_i/\delta$, backed out using eq 6, are plotted against the component loading at the upstream face of the membrane, *qi*,up. For all gases, the $\rho \mathbf{D}_i / \delta$ appears to increase with $q_{i, \text{up}}$. For CO₂, a slight maximum is also detectable at an upstream loading of 6 mol kg⁻¹ beyond which the $\rho \mathbf{D}_i / \delta$ appears to decrease with increasing *qi*,up.

The values of transport coefficients $\rho \mathbf{D}_i / \delta$ shown in Figure 4a,b do not reflect the true M-S diffusivities \mathbf{P}_i because they are based on eqs 6 and 7, which are derived assuming \mathbf{P}_i is independent of loading q_i . The proper procedure for backing

Figure 2. The experimental data on unary permeation fluxes for He and H_2 are plotted against the driving force DF_i , calculated from eq 8 for He or eq 9 for H_2 . The straight lines have been drawn taking the fitted value of $\rho \mathbf{D}/\delta = 0.322$ and 0.293 kg m⁻² s⁻¹ for He and H₂, respectively.

Figure 3. The plots of N_i vs DF_i for (a) CO_2 , N_2 , O_2 , CO , Ar , and (b) CH4.

out the M-S diffusivities is to first establish the correct $\mathbf{P}_i - q_i$ dependence. To quantify the loading-dependence, we use the model developed by Reed and Ehrlich;^{21,22,29} this model gives the following expression

$$
\mathbf{D}_{i} = \mathbf{D}_{i}(0) \frac{(1 + \epsilon_{i})^{z-1}}{(1 + \epsilon_{i}/\phi_{i})^{z}}
$$
(10)

Figure 4. Transport coefficients $\rho \mathbf{P}_i / \delta$ backed out using eq 6 (a) CO₂, N2, O2, CO, Ar, and (b) CH4.

where *z* is the coordination number, representing the maximum number of neighbors within a cage; we take $z = 5$ (i.e., one less than the maximum capacity of molecules that can be accommodated in each cage). The other parameters are defined as (see Krishna et al.²¹ for more detailed discussions and derivations)

$$
\epsilon_i = \frac{(\beta_i - 1 + 2\theta_i)\phi_i}{2(1 - \theta_i)}; \quad \beta_i = \sqrt{1 - 4\theta_i(1 - \theta_i)(1 - 1/\phi_i)};
$$

$$
\theta_i = \frac{q_i}{q_{i, \text{sat}}} \tag{11}
$$

With the Reed-Ehrlich loading-dependence, we may define a modified driving force *MDFi*

$$
MDF_i \equiv \int_{f_{i,\text{down}}}^{f_{i,\text{up}}} \frac{(1+\epsilon_i)^{z-1}}{(1+\epsilon_i/\phi_i)^z} \frac{q_i}{f_i} df_i \tag{12}
$$

The integration must be carried out numerically. The modified driving force must be expected to bear a linear relation with the permeation flux

$$
N_i = \frac{\rho \mathbf{D}_i(0)}{\delta} M D F_i \tag{13}
$$

The plots for CO_2 , N_2 , CO , O_2 , Ar , and CH_4 are shown in Figure 5a-c. The Reed-Ehrlich parameter ϕ_i was chosen to yield a straight line relationship; the best fit values of ϕ_i that yield linear

Figure 5. The experimental data on unary permeation fluxes of (a) $CO₂$, (b) N₂, CO, O₂, Ar and (c) CH₄ is plotted against the modified driving force *MDFi*, calculated from eq 12, taking the parameter values specified in Table 2. The straight lines are drawn using eq 13 with fitted values of $\rho \mathbf{D}_i(0)/\delta$ specified in Table 2.

dependencies of *Ni* on *MDFi* are listed in Table 2 along with the corresponding zero-loading transport coefficients $\rho \mathbf{D}_i(0)/\delta$.

To rationalize the values of the zero-loading transport diffusivities and Reed-Ehrlich parameter ϕ_i , we need information on the molecular dimensions. Figure 6 presents drawings showing approximate molecular dimensions of the various gases used in these investigations; these dimensions were estimated using published force fields for molecule-molecule interactions.³⁰⁻³² The values of the zero-loading transport coefficients $\rho \mathbf{D}_i(0)/\delta$ appear to decrease sharply as the molecular diameter approaches the window size, which has a value of 3.8 Å for SAPO-34; see Figure 7a. For CO_2 , N₂, CO, and Ar, the molecular diameter is in the 3-3.4 Å range and the $\rho \partial_i(0)/\delta$ appear to decrease with increasing molecular length; see Figure 7b. The loading-

Figure 6. Drawing showing the approximate molecular dimensions of He, H₂, CO₂, N₂, O₂, CO, Ar, and CH₄.

TABLE 2: Reed-**Ehrlich Parameters in Various Zeolite Structures**

		Reed-Ehrlich model parameters in eqs 10 and 11		
molecule	$\rho \mathbf{D}_i(0)/\delta$	Z.		
He	0.322			
H ₂	0.293			
CO ₂	4.6×10^{-3}		2.1	
CH ₄	3.2×10^{-4}	5	3	
N_2	1.07×10^{-2}	5	2.3	
O ₂	2.53×10^{-2}	5	1.8	
C _O	1.14×10^{-2}	5	2	
Ar	1.37×10^{-2}			

dependence is stronger when the molecular diameter is larger; see Figure 7c. For an interpretation of the loading-dependence of the M-S diffusivity in terms of the free energy barrier for hopping of molecules between cages, see the papers of Beerdsen et al.18,33

4. Modeling of Mixture Permeation

The M-S equations^{13,34,35}

$$
- \rho \frac{q_i}{RT} \frac{d\mu_i}{dx} = \sum_{\substack{j=1 \ j \neq 1}}^n \frac{q_j N_i - q_i N_j}{q_{j, sat} \theta_{ij}} + \frac{N_i}{\theta_i}; \quad i = 1,...n \quad (14)
$$

are widely used for modeling and interpretation of zeolite membrane permeation experiments for a variety of molecules and mixtures.^{3,36,37} In eq 14, Φ _{*i*} is the M-S diffusivity of species i in the mixture, q_i is the molar loading, $q_{i, \text{sat}}$ is the saturation capacity of species i , \mathbf{D}_{ij} are the binary exchange coefficients. The major advantage of using the M-S equations is that the diffusion coefficients \hat{D}_i and \hat{D}_{ij} can be estimated from pure component adsorption and diffusion data.13,35 Zhu et al.38 and Li et al.²³ have modeled permeation of $CO₂$ (1)-CH₄ (2) mixtures across MFI and SAPO-34 membranes, respectively by estimating that the binary exchange parameter \mathbf{P}_{ij} using the interpolation formula.

$$
\mathbf{D}_{ij} = [\mathbf{D}_{i}]^{q/(q_i + q_j)} [\mathbf{D}_{j}]^{q/(q_i + q_j)}
$$
(15)

On the basis of MD simulations of diffusion in $CO₂-CH₄$ mixtures, Krishna et al.^{21,32} have argued that in zeolite structures that consist of cages separated by narrow windows, such as CHA and DDR, intercage jumps are uncorrelated and that as an

Figure 7. Dependence of (a) the zero-loading transport coefficient $\rho \hat{D}_i(0)/\delta$ on the molecular diameter, (b) the zero-loading transport coefficient $\rho \mathbf{D}_i(0)/\delta$ on the molecular length, and (c) the Reed-Ehrlich parameter ϕ_i on the molecular diameter.

approximation, the binary exchange coefficient \mathbf{b}_{12} can be taken to be infinite, that is

$$
N_i = -\rho \mathbf{D}_i \frac{q_i}{RT} \frac{\mathrm{d}\mu_i}{\mathrm{d}x}; \quad i = 1,...n \tag{16}
$$

SAPO-34 is an isotype of CHA and the same approximation may be expected to hold. The gradient of the chemical potentials in eq 16 can be related to the gradients in the loadings by defining a matrix of thermodynamic factors [Γ]

$$
\frac{q_i}{RT}\frac{d\mu_i}{dx} = \sum_{j=1}^n \Gamma_{ij} \frac{dq_j}{dx}; \quad \Gamma_{ij} = \frac{q_i}{f_i} \frac{\partial f_i}{\partial q_j}; \quad ij = 1,..2 \quad (17)
$$

The elements Γ_{ij} in eq 17 can be estimated using the idealadsorbed solution theory of Myers and Prausnitz³⁹ and the fits

Figure 8. Comparison of transport coefficients, $\rho \mathbf{P}_i / \delta$, backed out from unary permeation and binary mixtures containing (a) $CO₂/CH₄$, (b) CO_2/N_2 , (c) N₂/CH₄, (d) CH₄/H₂, (e) N₂/H₂, and (f) CO/H₂.

of the pure component isotherms. Integrating eq 17 across the membrane thickness we obtain

$$
N_i = \frac{\rho}{\delta} \int_{\text{upstream}}^{\text{downstream}} \mathbf{D}_i \sum_{j=1}^n \left(\Gamma_{ij} \frac{\mathbf{d}q_j}{\mathbf{d}x} \right) \mathbf{d}x; \quad i = 1, \dots n \quad (18)
$$

The details of the numerical procedures for calculation of the fluxes are described in earlier work.3,40

Before proceeding with the estimation of the fluxes N_i using the pure component data on the transport coefficients listed in Table 2, we shall demonstrate the applicability of the uncoupled M-S equations (18) by backing out the transport diffusivity $\rho \partial_i / \delta$ as a function of the total upstream loading, $q_{up} = q_{1,up}$ $+$ $q_{2,\text{up}}$, for the six binary mixture permeation experiments using

$$
\frac{\rho \mathbf{D}_i}{\delta} = \frac{N_i}{\int_{\text{upstream}}^{\text{downstream}} \sum_{j=1}^n \left(\Gamma_{ij} \frac{\mathbf{d}q_j}{\mathbf{d}x} \right) \mathbf{d}x}; \quad i = 1,...n \quad (19)
$$

The results are presented in Figure 8a–f, along with the backed-out transport coefficients from unary permeation experiments (reported in Figure 4a,b). For the (a) CO_2/CH_4 , (b) $CO_2/$ N₂, and (c) CH₄/N₂ mixtures, we note that the $\rho \partial$ *i*/ δ backed out either from pure component or mixture permeation experiments exhibit similar loading-dependences and correspond closely to each other. This confirms that the assumption of

Figure 9. Permeation fluxes for binary (a) CO_2/CH_4 , (b) CO_2/N_2 , and (c) N2/CH4 mixtures as function of the upstream total pressure, *f*up. The continuous solid lines represent the calculations by numerical solution of eq 18.

infinite value for the binary exchange parameter \mathbf{b}_{12} is a good one. A slightly different picture emerges for the mixture containing H₂. From the $\rho \mathbf{D}_i / \delta$ data for CH₄/H₂, N₂/H₂, and $CO/H₂$ mixtures (see Figure 8d-f), there appears to be a slight slowing-down of the more mobile H_2 with a concomitant and small speeding-up of the more tardy species, $CH₄$, $N₂$, and CO in the three binary mixtures, respectively. This would indicate that the assumption of an infinite value of \mathbf{b}_{12} is not as good for mixtures with hydrogen and must be viewed only as an approximation.

It now remains to compare the estimate of the permeation fluxes in the mixture from eq 18 with experimental values. The

Figure 10. (a) Permeation fluxes and (b) permeation selectivities α_{perm} , for ternary $CO_2/CH_4/N_2$ mixture as a function of the upstream total pressure, *f*up. The continuous solid lines represent the calculations by the numerical solution of eq 18 across the membrane. The dashed lines represent integration of eqs 14 and 15 across the membrane.

M-S diffusivity \hat{D}_i of component *i* in the mixture is estimated on the basis of the total mixture occupancy, θ , defined by

$$
\theta = \sum_{i=1}^{n} \frac{q_i}{q_{i, \text{sat}}} \tag{20}
$$

and the Reed-Ehrlich model, eq 10, and the pure component transport data in Table 2. Figure $9a-c$ shows that the estimations of N_i using eq 18 for (a) CO_2/CH_4 , (b) CO_2/N_2 , and (c) CH_4/N_2 mixtures are in very good agreement with experimental values (filled solid symbols) for the entire range of upstream pressure conditions. This result is to be expected in view of the good agreement in the backed-out transport coefficients $\rho \partial_i/\delta$ of mixtures with that of pure component data; cf. Figure $8a-c$.

Figure 10a shows the experimentally determined fluxes for the ternary $CO_2/CH_4/N_2$ mixture as a function of the upstream total pressure. The predictions of the M-S mixture model (eq 18), shown by the continuous solid lines in Figure 10a, are in good agreement with experiment. Figure 10b shows the experimentally determined permeation selectivity α_{perm} defined by

$$
\alpha_{\text{perm}} = \frac{N_1/N_2}{f_{1,\text{up}}/f_{2,\text{up}}}
$$
 (21)

for CO_2/CH_4 and N_2/CH_4 separation in the ternary mixture. Inclusion of the binary exchange coefficients \mathbf{b}_{ij} , estimated

Figure 11. Permeation fluxes for binary (a) CH_4/H_2 , (b) N_2/H_2 , and (c) CO/H2 mixtures as function of the upstream total pressure, *f*up. The continuous solid lines represent the calculations by numerical solution of eq 18 across the membrane.

using eq 15 in the M-S equation (14), will have the effect of slowing-down the "faster" molecules, while concomitantly speeding-up the "tardier" molecules. These predictions are shown by the dashed lines in Figure 10b; for both mixtures this leads to a significantly poorer match with experiments. For $CO₂/CH₄$ and N₂/CH₄ mixtures, allowance for a finite binary exchange coefficient \mathbf{P}_{ij} predicts a significantly lower α_{perm} than in the experiments because in both these cases we have slowingdown of the more mobile $CO₂$, or $N₂$, and concomitant speedingup of the tardy CH4. The conclusion to be drawn from the results presented in Figure 10b is that intercage hopping of molecules $CO₂$, CH₄, and N₂ within SAPO-34 occurs independent of one other and validates the use of the simplified M-S equation (18). There is no perceptible slowing-down or speeding-up process. The window size of SAPO-34 is of the order of 3.8 Å, and only one molecule can pass through the window at a time. The intercage hopping of molecules between cages is uncorrelated, and the three binary exchange coefficients \hat{P}_{ij} all have large values.

We now turn to permeation of the mixture containing H_2 ; the measured fluxes for binary (a) CH_4/H_2 , (b) N_2/H_2 , and (c) CO/H2 mixtures with equimolar feed mixture compositions are shown in Figure $11a-c$ (filled solid symbols) as a function of the total upstream pressure, *f*up. The model calculations are indicated by continuous solid lines in Figure 11. For all three binary mixtures, the fluxes are predicted with reasonable accuracy using only pure component diffusivity and isotherm data. We note, however, that for all three mixtures the fluxes of H2 are slightly overestimated; the reason for this is that there is a slight slowing-down of H_2 in the mixture due to the presence of the tardy companion species (CH₄, N_2 , or CO) as was stressed earlier; cf. Figure 8d-f. We see also note that the estimation of the fluxes of CH_4 , N_2 , and CO are slightly lower than found experimentally; this is due to the slight speeding-up of these tardier components. The assumption of infinite exchange \mathbf{b}_{12} is to be considered as a reasonable, but not perfect, approximation for hydrogen containing mixtures.

5. Conclusions

The following conclusions can be drawn from the results presented in this paper.

(1) The M-S diffusivities $\hat{\mathbf{p}}_i$ of He, H₂, CO₂, N₂, O₂, CO, Ar, and CH4 in SAPO-34 have been backed out using unary permeation data. While the assumption of a loading-independent \mathbf{P}_i is valid for He and H₂, the M-S diffusivities of CO₂, N₂, O₂, CO, Ar, and CH4 are all significantly loading-dependent.

(2) The model of Reed-Ehrlich provides a convenient practical procedure for modeling the loading-dependence of \mathbf{D}_i ; the severity of the loading-dependence is captured in this model by the parameter ϕ_i . The larger the diameter is of the molecule, the stronger the loading-dependence is.

(3) Binary CO_2/CH_4 , CO_2/N_2 , CH_4/N_2 , and ternary $CO_2/CH_4/$ N2 mixture permeation in SAPO-34 is best modeled by the M-S equation (18) wherein the intercage hopping of molecules is taken to be independent of one another. This corresponds with the assumption of an infinite exchange coefficient \mathbf{P}_{ij} .

(4) For permeation of binary mixtures H_2/CH_4 , H_2/N_2 , and H_2/CO , the assumption of infinite exchange coefficient \mathbf{D}_{12} is not a perfect one; there is a slight slowing-down of mobile H_2 with concomitant speeding-up of the tardier CH_4 , N_2 , or CO. Even so, the estimations of the fluxes using eq 18 are in reasonably good agreement with experiment.

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Supporting Information Available: This includes details of membrane synthesis, characterization, and measurement procedures along with the experimental data. This material is available free of charge via the Internet at http://pubs.acs.org/.

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Interpreting unary, binary and ternary mixture permeation across SAPO-34 membrane with loading dependent Maxwell-Stefan diffusivities

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Experimental Methods and Data

1. Experimental Methods

Membrane Synthesis

SAPO-34 membranes were synthesized by seeding on porous stainless steel tubes (0.8-μm pores, Pall Corp.). The detailed synthesis procedure was given elsewhere.¹ Non-porous, stainless steel tubes were welded onto each end of supports. The permeate area was approximately 7.8 cm^2 . The synthesis gel composition for SAPO-34 seeds was 1.0 Al₂O₃: 1.0 P₂O₅: 0.3 SiO₂: 1.2 TEAOH: 55 H₂O. This composition is different to that used in our previous published study.²

The gel was prepared by stirring $H_3PO_4(85 \text{ wt\%}$ aqueous solution, Aldrich), $Al(i-C_3H_7O)_3$ (> 99.99%, Aldrich), and distilled H_2O at room temperature for 12 h. Then the template, tetra-ethyl ammonium hydroxide (TEAOH, 35 wt% aqueous solution, Aldrich), was added and the mixture stirred for 30 min before the colloidal silica sol (Ludox AS40, 40% aqueous solution, Aldrich) was added to the mixture. The solution was sealed, stirred, and aged for approximately 24 h at room temperature. Next, the gel was added to a Teflon-lined autoclave. The hydrothermal synthesis was carried at 493 K for about 24 h. After synthesis, the powder was collected and washed with distilled water at 295 K for three times, and then dried at 373 K in a vacuum oven for 2 h. The powder was calcined at 823 K for 10 h. The heating and cooling rates were 6 K/min. The calcined crystals were used as seeds for membrane preparation.

Before membrane synthesis, the supports were boiled in distilled water for 3 h and dried at 373 K under vacuum for 30 min. The stainless steel tubes were then wrapped with Teflon tape on their outside. The tubes were then treated by rubbing the inside surface of the support tube with dry SAPO-34 particles. The seeded tubes were placed in an autoclave, which was then filled with the synthesis gel that has the same composition as that used for seeds. The hydrothermal synthesis was carried at 493 K for about 24 h. After synthesis, the membrane was washed with distilled water at 295 K and dried at 373 K in a vacuum oven for 2 h. A second synthesis step was applied using the same procedure, but the tube

was inverted to obtain a more uniform layer. The membranes were calcined in air at 663 K for 10 h. The heating and cooling rates were 0.6 and 0.9 K/min, respectively.

Characterization

Crystals collected during membrane synthesis were calcined at 823 K for 10 h, and then used in various analyses including adsorption measurements. X-ray diffraction (XRD, Scintag PAD-V diffractometer) confirmed the CHA structure. The chemical composition of the crystals determined by inductively coupled plasma analysis (Varian UltraMass 700 inductively coupled plasma mass spectrometer with a CETAC LXS-200+ laser ablation system) was $(Si_{0.061}Al_{0.483}P_{0.455})O_2$.

Adsorption experiments were carried out in an Autosorb-1 (Quantachrome Corp. Model AS1-C-VP-RGA) system. Prior to each adsorption experiment, the sample was outgassed in vacuum at 493 K for at least 2 h. Adsorption isotherms for H_2 , CO_2 , CH_4 , N_2 , O_2 , Ar, and CO were measured at 295 K.

Nitrogen adsorption was performed at 77 K to determine the BET surface area and micropore volume. The SAPO-34 zeolite had a BET surface area of 500 m²/g and a micropore volume of 0.26 cm³/g.

Permeation experiments

Unary, binary and ternary mixture permeations were measured in the flow system described previously;² the system was modified to operate up to 7.2 MPa pressure. The membrane was mounted in a stainless steel module and sealed at each end with silicone o-rings. The pressure on each side of the membrane was independently controlled. Unary and mixture permeations were investigated as functions of feed pressure while maintaining a constant permeate pressure of 84 kPa. Fluxes were measured using a soap film bubble flow meter. The system was leak checked by replacing the membrane with a solid stainless steel tube. The leak rate for a 7-MPa pressure drop across the O-ring was less than 0.01% of the measured CH4 flux for a 50/50 mixture at the same pressure drop across a membrane.

Figure 1 shows a schematic of the membrane set-up.The compositions of the feed, retentate, and permeate streams were measured online using a SRI 8610 C GC gas chromatograph equipped with a thermal conductivity detector and HAYESEP-D column (Alltech). The oven, injector, and detector temperatures were all kept at 423 K.

2. Permeation data

The experimental data for unary permeation of $CO₂ CH₄$, N₂, He, H₂, O₂, CO and He are presented in Tables 1, 2 3, 4, 5, 6, 7, and 8. The binary permeation data N_2/H_2 , CH_4/H_2 and CO/H_2 mixtures are presented in Tables 9, 10 and 11. The binary permeation data for CO_2/CH_4 , CO_2/N_2 , and N_2/CH_4 mixtures are summarized in Tables 12, 13, and 14. The ternary permeation data for $CO₂/N₂/CH₄$ mixtures are presented in Table 15. The gas phase compositions of the retentate (upstream) and permeate (downstream) compartments were also measured and on this basis the partial pressures on both compartments were determined. The *average* (logarithmic mean) values of the upstream partial pressures, $f_{\text{i,up}}$, are reported in Tables 9, 10, 11, 12, 13, 14, and 15 were used. For purposes of model calculations to compare with experiments, these precise partial pressures as specified in the second and third columns of Tables 9, 10, 9, 10, 11, 12, 13, 14, and 15 were used.

For plotting purposes the total upstream pressures, listed in the first column in Tables 9, 10, 11, 12, 13, 14, and 15 are employed on the x-axis in the main text of this paper.

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Table 4. Unary He permeation data.

Table 5. Unary H₂ permeation data.

Table 8. Unary Ar permeation data.

Table 9. Binary N_2-H_2 permeation data

Upstream	N ₂	H ₂	N ₂	H ₂	N_2 flux	H_2 flux
total	upstream	upstream	downstream	downstream	N_1	N_2
pressure	partial	partial	partial	partial		
$f_{\rm up}$	pressure	pressure	pressure	pressure		
	1 _{up}	$f_{2,\text{up}}$	$f_{1,\text{down}}$	$f_{2,\text{down}}$		
0.77	0.39	0.38	0.0204	0.0636	0.0099	0.031
1.46	0.74	0.72	0.0215	0.0625	0.0205	0.060
2.84	1.46	1.38	0.0238	0.0602	0.0387	0.098
4.22	2.19	2.03	0.0260	0.0580	0.0574	0.128
5.60	2.91	2.69	0.0285	0.0555	0.0763	0.149
6.97	3.63	3.35	0.0299	0.0541	0.0911	0.165

Table 10. Binary CH₄ -H₂ permeation data

Upstream	CH ₄	H ₂	CH ₄	H ₂	CH ₄ flux	H_2 flux
total	upstream	upstream	downstream	downstream	N_1	N_2
pressure	partial	partial	partial	partial		
$f_{\rm up}$	pressure	pressure	pressure	pressure		
	$f_{1,\text{up}}$	$f_{2,\text{up}}$	$f_{1,\text{down}}$	$f_{2,\text{down}}$		
0.77	0.39	0.38	0.0054	0.0786	0.0016	0.024
1.46	0.75	0.71	0.0067	0.0773	0.0035	0.040
2.84	1.47	1.37	0.0084	0.0756	0.0067	0.060
4.22	2.20	2.01	0.0105	0.0735	0.0105	0.073
5.60	2.94	2.66	0.0117	0.0723	0.0133	0.083
6.97	3.67	3.31	0.0140	0.0700	0.0176	0.088

Table 11. Binary CO-H₂ permeation data.

Total	CO ₂	CH ₄	CO ₂	CH ₄	CO ₂ flux	CH ₄ flux
upstream	upstream	upstream	downstream	downstream	N_1	N_2
pressure	partial	partial	partial	partial		
$f_{\rm up}$	pressure	pressure	pressure	pressure		
	1 _{up}	$f_{2,\text{up}}$	$f_{1,\text{down}}$	$f_{2,\text{down}}$		
0.77	0.37	0.40	0.083	0.0013	0.064	0.0010
1.46	0.69	0.77	0.083	0.0014	0.102	0.0017
2.50	1.14	1.35	0.083	0.0014	0.140	0.0024
3.88	1.72	2.14	0.082	0.0017	0.171	0.0035
5.26	2.27	2.95	0.082	0.0018	0.196	0.0044
6.70	2.82	3.83	0.082	0.0020	0.221	0.0055
7.19	3.00	4.13	0.082	0.0021	0.228	0.0059

Table 12. Binary CO_2 -CH₄ permeation data. The feed composition was 50% : 50%.

Total	CO ₂	N ₂	CO ₂	N ₂	CO ₂ flux	N_2 flux
upstream	upstream	upstream	downstream	downstream	N_1	N_2
pressure	partial	partial	partial	partial		
$f_{\rm up}$	pressure	pressure	pressure	pressure		
	$f_{1,\text{up}}$	$f_{2,\text{up}}$	$f_{1,\text{down}}$	$f_{2,\text{down}}$		
0.77	0.39	0.38	0.079	0.005	0.065	0.0037
1.46	0.72	0.74	0.080	0.004	0.108	0.0059
2.15	1.04	1.11	0.079	0.005	0.136	0.0091
2.83	1.35	1.48	0.079	0.005	0.154	0.0107
3.52	1.66	1.86	0.078	0.006	0.168	0.0122

Table 13. Binary CO_2-N_2 permeation data. The feed composition was 52.7% : 47.3%.

Total	N ₂	CH ₄	N ₂	CH ₄	N_2 flux	CH ₄ flux
upstream	upstream	upstream	downstream	downstream	N_1	N_2
pressure	partial	partial	partial	partial		
$f_{\rm up}$	pressure	pressure	pressure	pressure		
	$f_{1,\text{up}}$	$f_{2,\text{up}}$	$f_{1,\text{down}}$	$f_{2,\text{down}}$		
0.773	0.386	0.388	0.079	0.005	0.065	0.0037
1.463	0.728	0.735	0.080	0.004	0.108	0.0059
2.152	1.069	1.084	0.079	0.005	0.136	0.0091
2.842	1.409	1.433	0.079	0.005	0.154	0.0107
3.531	1.748	1.783	0.078	0.006	0.168	0.0122

Table 14. Binary N_2 –CH₄ permeation data. The feed composition was 50% : 50%.

Table 15. Ternary CO_2 - $CH_4 - N_2$ permeation data.

4. Captions for Figures

