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Vibration-induced granular segregation in a pseudo-2D column: The (reverse) Brazil nut effect

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Abstract

The segregation of nine different disk-shaped intruders in a pseudo-2D granular bed consisting of 0.85 mm polystyrene beads under the influence of vertical vibrations was studied. The intruders used were all fabricated of brass and were much denser than the polystyrene particles. Depending on the vibration amplitude and frequency, intruders were able to segregate to the top or bottom of the granular bed as long as the vibration intensity Γ was greater than 1. At a fixed vibration amplitude, upward segregation, also known as the Brazil nut effect, occurred at a much higher frequency than downward segregation, which is also known as the reverse Brazil nut effect. Segregation to the top was always much quicker than segregation to the bottom irrespective of size and mass of the intruder used. By the use of high-speed video movies, it was found that the disk-shaped intruders rose along the bed height by a void filling mechanism.

Keywords: Granular segregation; Granular bed; Brazil nut effect; Reverse Brazil nut effect; Vibrating bottom

1. Introduction

In recent years, a significant amount of attention in scientific literature has been given to the Brazil nut effect [1–7]. Simply explained, this is a phenomenon whereby when a mixture of different sized nuts is vibrated, the largest nuts rise to the top. This phenomenon holds true not just for nuts, but also for a wide range of granular mixtures. It is not simply the counterintuitive nature of the Brazil nut effect that makes it interesting but the potential applications to which it could be put to use. For instance, industries dealing with the handling and separation of granular materials and other forms of particulate matter would benefit from a well-founded scientific understanding of this effect [8–10].

A general agreement in the literature about the Brazil nut effect is that, under the influence of vibrations, a mixture of different sized granular materials tends to segregate. Depending on the vibration amplitude and frequency, the larger particles may segregate to the top or bottom, with these phenomena known as the Brazil nut effect (BNE) and reverse Brazil nut effect (RBNE), respectively. This, however, is as far as any form of agreement found in the literature goes. Under what conditions the Brazil nut and reverse Brazil nut effects occur is still hotly disputed [11], as well as the fundamental mechanisms responsible for the occurrence of these phenomena. Due to the large number of variables involved in systems where the BNE and RBNE occur, not a single generally applicable model capable of quantitatively predicting the behaviour of large particles in a vibrated granular bed is available. For instance, Shinbrot and Muzzio [1] discovered that in a vertically vibrated granular bed, large heavy intruders move to the surface while light intruders move to the bottom at "large" amplitudes of vibration. Yan et al. [12] observed that the RBNE only occurs when the intruder density is less than that of the surrounding bed particles. This however is in contrast to the findings of Breu et al. [4] which showed that the RBNE only occurs when the density of the intruder was greater than that of the surrounding bed particles. Huerta and Ruiz-Suárez [11,13] attempted explaining the problem from another viewpoint by proposing that at high frequencies, the granular bed fluidizes and while light particles rise due to buoyancy, heavy particles sink. The rise time of a large particle also depends on the ratio of its

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density to that of the surrounding particles, ρ/ρ_b. The resulting trend has been observed to be non-monotonic by a number of researchers [11,14] with the maximum rise time occurring when at $\rho/\rho_b \approx 0.5$.

The aim of the work presented in this paper is the experimental study of the BNE and RBNE. Until now, most of the studies on these effects have been conducted within a restricted range of vibration amplitudes and frequencies. This study, which utilizes disk-shaped intruders placed in a pseudo-two-dimensional (2D) granular bed, focuses on a wide range of vibration amplitudes and frequencies, attempting to demonstrate that both the BNE and RBNE can be obtained with the same intruder in a granular bed. The intruders used in this study differ in diameter, mass and density in an attempt to understand how a variation of these parameters affects the rise or sink behaviour of an intruder in a granular bed. The decision to use a pseudo-2D granular bed rather than a 3D bed was guided by the need to be able to fully observe the rise or sink motion of the intruder. While bed circulation patterns in a pseudo-2D granular bed will differ from those in 3D beds, experimental data obtained from the pseudo-2D bed would be very useful for understanding the underlying mechanisms that drive the BNE and RBNE.

2. Experiment

Experiments were carried out in a pseudo-2D polyacrylate container setup shown in Fig. 1(a). The container was 0.2 m high, 0.15 m wide and 0.0045 m deep; see Fig. 1(b) for a close-up on the granular bed. It was filled with polystyrene beads to a height of 0.06 m measured from its base. The polystyrene beads had an average diameter of 0.85 mm and a density of 1080 kg/m^3. The pseudo-2D polyacrylate container was mounted on an air-cooled vibration exciter (TIRAvib 5220, Germany), which was used to generate sine wave displacements at a range of amplitudes, λ, and frequencies, f. In this report, the amplitude of vibration, λ, is defined as the absolute value of the maximum positive or negative displacement of the vibrating bottom from its rest position. The vibrating bottom was coupled to a power amplifier, allowing it to be operated within a frequency range of 2 to 5000 Hz. Depending on the operating frequency, the amplitude of vibration could be varied in the range of 0 to 12 mm. The vibrating system was fully automated and controlled from a personal computer (PC) using Signal Calc 550 Vibration Controller software (Data Physics Corporation, USA).

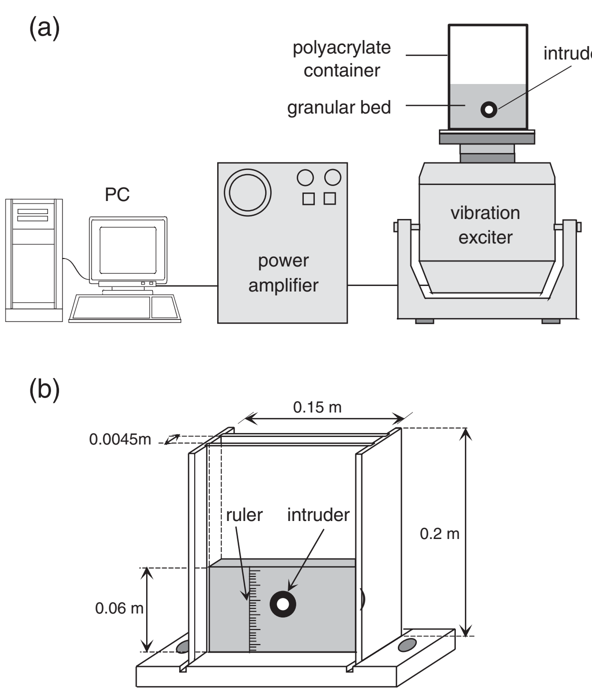

Fig. 1. (a) Schematic representation of the experimental setup. (b) Close-up of the granular bed.

Table 1
Characteristics of the intruders used in this study

Intruder	Outer diameter, D (mm)	Inner diameter, d (mm)	Volume, V (10^{-6} m^3)	Mass, m (10^{-3} kg)	Apparent density, ρ (kg/m^3)
A1	5	0	0.085	0.713	8350
A2	10	0	0.342	2.853	8350
A3	10	6	0.342	1.826	5344
A4	15	0	0.769	6.419	8350
A5	15	11.2	0.769	2.840	3695
A6	20	0	1.367	11.411	8350
A7	20	12.1	1.367	7.234	5294
A8	20	17.32	1.367	2.853	2088
A9	25	0	2.135	17.830	8350

Disk-shaped brass intruders A1 – A9, nine in number, of different inner, d, and outer diameters, D, were used in the experiments carried out. The characteristic dimensions of these intruders are given in Table 1. Fig. 2 shows a sample representation using two different intruders. The drilling of holes of diameter d on certain intruders was necessary to vary the intruder density. All intruders were 4.35 mm thick, slightly less than the 4.5 mm depth of the polyacrylate container. Thus, the polystyrene background particles could not slip between the wall of the polyacrylate container and the intruder.

The motion of the intruder in the polystyrene granular bed was studied from video movies made using a Panasonic DSP color video camera, which was positioned in front of the granular bed. Movies were made with the camera at a capture rate of 25 frames per second (fps) and stored on a PC for analysis. In addition to these, high-speed video movies were also made using a Photron Fastcam-ultima 40K high-speed video camera at a capture rate of 750 fps in order to understand

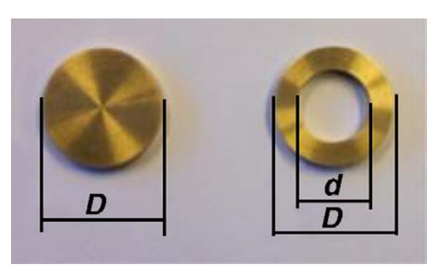

Fig. 2. Sample representation of the intruders used in our experiments.

the underlying mechanism responsible for the rise or sink of the intruders in the granular bed. At the start of each experiment, the granular bed was filled to the 0.06 m height with polystyrene beads. For sink experiments from the top of the bed, the intruder was partially buried in the granular bed, such that the top of the intruder corresponded to the top of the bed. For rise experiments from the bottom of the bed, the intruder was pushed to the bed bottom using a metal rod. While doing this, the bed was vibrated making it easier to get the intruder to the bottom. At the start of sink experiments from the top of the bed, and rise experiments from the bottom of the bed, the vibration exciter was always at rest. Certain rise and sink experiments were carried out (see Fig. 5) in which the starting height of the intruder varied along the bed height. For these experiments, the vibration exciter was always in operation, making it easy to alter the initial intruder height along the bed height using a metal rod.

For different vibration amplitudes, the minimum frequency at which an intruder sank from the top of the granular bed to the bottom, otherwise known as the sink frequency, f_{sink}, was determined. The minimum frequency at which an intruder rose from the bottom of the granular bed to the top, i.e. the rise frequency, f_{rise}, was also determined. Note that the intruder was assumed to be at the top of the granular bed when the top of the intruder matches the top of the granular bed. To obtain a detailed picture of the intruder movement, additional experiments were conducted at constant amplitude. By varying the frequency of vibration, regions in the granular bed in which the intruders sink and rise were determined. More information on the experimental setup used as well as video recordings and demonstrations are available at our website [15].

3. Results and discussion

Fig. 3 shows the results of a set of experiments carried out, using the four intruders A2, A3, A6 and A7 for which f_{rise} and f_{sink} were determined at seven different amplitude values,

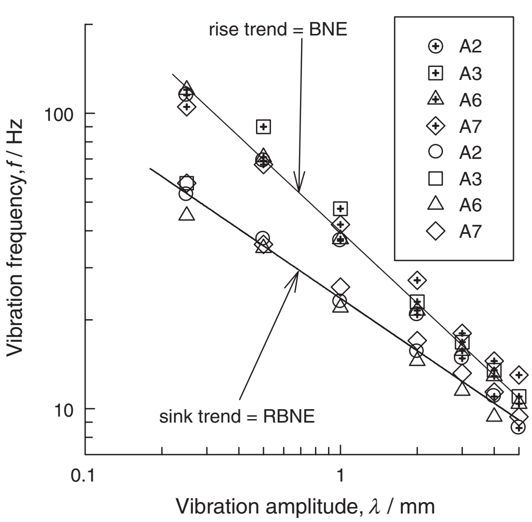

Fig. 3. The minimum vibration frequency as a function of amplitude, λ, at which an intruder rises from the bottom to the top of the granular bed, f_{rise} (cross-haired symbols) and sinks from the top to the bottom of the granular bed, f_{sink} (open symbols). Trend lines are drawn to guide the eye.

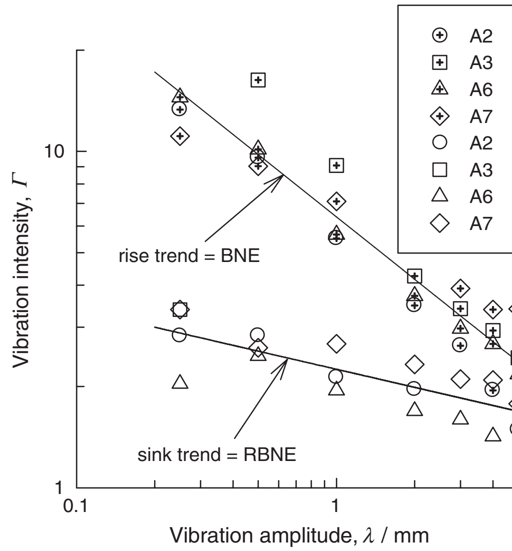

Fig. 4. The minimum vibration intensity, Γ, as a function of amplitude, λ, at which an intruder rises from the bottom to the top of the granular bed (cross-haired symbols); sinks from the top to the bottom of the granular bed (open symbols). Trend lines are drawn to guide the eye.

λ = 0.25, 0.5, 1, 2, 3, 4 and 5 mm. Two observations can clearly be made from this plot. Firstly, for a given amplitude, an intruder can be made to rise or sink simply by changing its frequency, with the rise frequency being consistently higher than the sink frequency. Secondly, as the amplitude of vibration λ is increased, both the f_{rise} and f_{sink} values decrease and approach each other. The plot in Fig. 4 indicates the vibration intensity, Γ corresponding to the BNE and RBNE in relation to the vibration amplitude. The vibration intensity is a dimensionless quantity widely used in the literature for quantifying the acceleration experienced by a granular mixture. It is defined as $\Gamma=\lambda(2\pi f)^2/g$ where g is the gravitational acceleration. As shown in Fig. 4, getting the intruders to rise from the bottom of the granular bed to the top requires an almost exponential increase in the vibration intensity with decreasing amplitude. This contrasts with the slight increase in vibration intensity for getting the intruders to sink as the amplitude of vibration is reduced. For a given amplitude, Figs. 3 and 4 show some spread in the sink and rise frequencies for different intruders.

The findings in Figs. 3 and 4 are interesting in that they make it clear that both the BNE and RBNE can be made to occur by simply altering the operating parameters. Although both the BNE and RBNE have been observed in experimental literature studies [1,11,12,14], most of these studies have focused on demonstrating the rise or sink time of an intruder as a function of the ratio of the intruder density to the density of the granular bed, ρ/ρ_b. To the best of our knowledge, not a single study has focused on the effect of a change in both the vibration amplitude and frequency on the rise and sink characteristics of a single intruder, making it difficult to directly relate the discoveries presented in this paper to already available information in the literature.

Fig. 4 illustrates that for vibration amplitudes $\lambda<1$ mm, the RBNE occurs for all intruders shown when $\Gamma<4$. As λ is

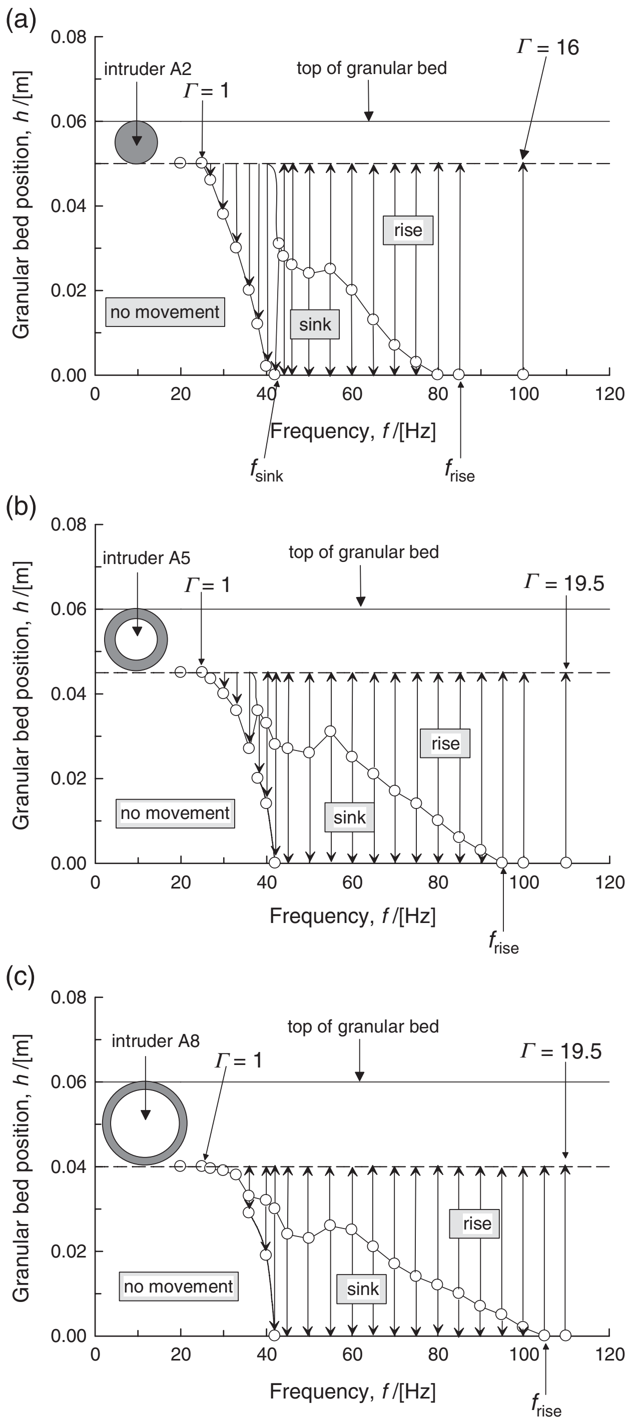

Fig. 5. Regions of rise and sink for intruders of the same mass but different volumes at an amplitude λ of 0.4 mm and varying frequencies: (a) intruder A2; (b) intruder A5; and (c) intruder A8.

increased, only the heaviest intruder particles, A6 and A7, continue to experience the RBNE. In the study of Shinbrot and Muzzio [1], RBNE was observed for $\rho/\rho_b<0.5$ using a mechanical shaker operated at $\Gamma=0.2$ and $f=7.1$ Hz (corresponding to $\lambda=0.99$ mm). The background particles used by Shinbrot and Muzzio [1] were 0.04 mm diameter spherical glass beads. Yan et al. [12] using 0.12 mm diameter glass beads in a granular bed also observed the RBNE for $\rho/\rho_b<0.5$ at $\Gamma=3$ and $f=30$ Hz (corresponding to $\lambda=0.83$ mm). Huerta and Ruiz-Suárez [11,13] showed that an increase in frequency from 5 Hz to 50 Hz leads to the transition of a heavy intruder's motion ($\rho/\rho_b>0.5$) from BNE to RBNE. They employed background particles of a comparable density to those used in this report, though their particles were three times as large. The frequency increase reported by Huerta and Ruiz-Suárez occurred at a constant vibration intensity Γ of 3, implying that λ decreased from 29.8 mm to 0.298 mm. The results of Huerta and Ruiz-Suárez are in agreement with the 2D experimental study in this report concerning the RBNE. However, since ρ/ρ_b was never less than 0.5 in this study, comparisons cannot be made with the findings of Shinbrot and Muzzio [1] and Yan et al. [12].

In Fig. 5(a), (b) and (c), regions of rise and sink for intruders A2, A5, and A8 of approximately the same mass (0.00285 kg) but

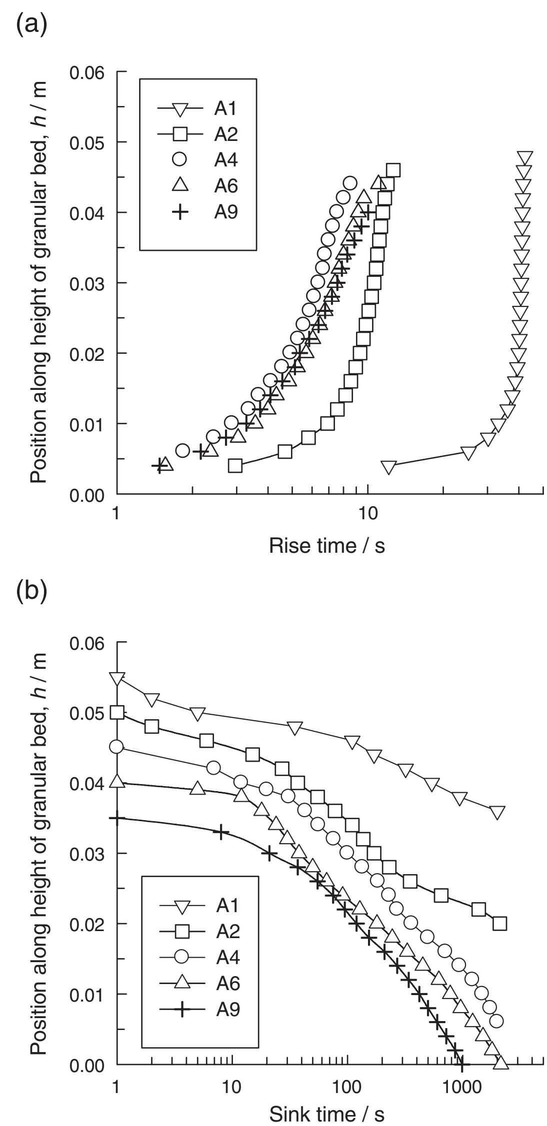

Fig. 6. (a) Intruder rise position h as a function of time at a frequency of 100 Hz and an amplitude of 0.4 mm. (b) Intruder sink position h as a function of time at a frequency of 36 Hz and an amplitude of 0.4 mm. Refer to Table 1 for the dimensions of the intruders.

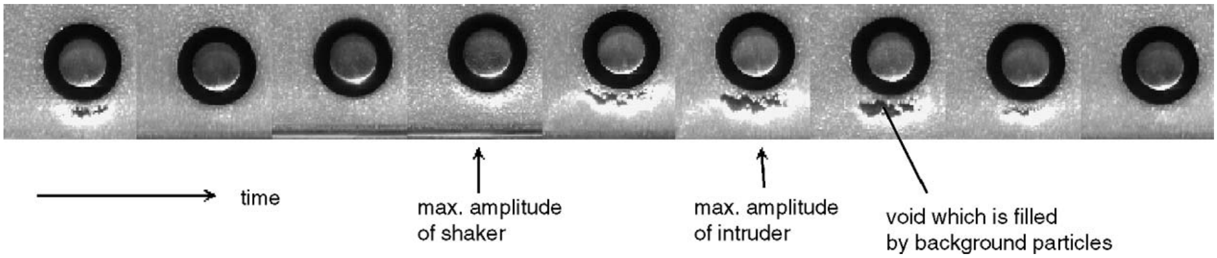

Fig. 7. A sequence of movie frames from the high-speed video camera for intruder A7. Vibration frequency f=22.3 Hz and vibration amplitude λ=2.5 mm. The time between frames is 5.33 ms. Frame size is 34 mm×34 mm.

different volumes, and apparent densities, are shown for a fixed vibration amplitude λ=0.4 mm and frequencies f varying from 20 Hz to 110 Hz. Below a frequency of 30 Hz, corresponding roughly to Γ=1, no intruder motion was observed, although the intruder starting positions were varied along the entire bed height. Vertical arrows are used to indicate the direction of intruder motion. A circle indicates a vertical position along the bed height corresponding to a point (1) above which an intruder only rises, indicated by upward arrows; (2) below which an intruder only sinks, indicated by downward arrows; (3) or at which an intruder comes to rest within the granular bed neither rising nor sinking, indicated by a downward arrow resting on the circle. While intruder A2 experienced both minimum rise and sink frequencies, intruders A5 and A8 only experienced minimum rise frequencies. In other words, at no frequency for the chosen amplitude λ=0.4 mm could intruders A5 and A8 be made to sink from the top of the granular bed to the bottom. The minimum rise frequency f_{rise} obtained corresponded to 85 Hz, 95 Hz and 105 Hz for intruders A2, A5 and A8, respectively, indicating that the larger the volume of an intruder, the larger the minimum frequency required for it to experience the Brazil nut effect in a granular bed.

The positions of five intruders, A1, A2, A4, A6 and A9, as functions of time were measured for the BNE and RBNE at an amplitude λ=0.4 mm. The results are shown in Fig. 6(a) and (b). Fig. 6(a) and (b) clearly demonstrate that both the mass of the intruder and its size (i.e. diameter) have an influence on the rise and sink behaviours, and that BNE proceeds much quicker than RBNE. A non-linearity is observed in both the rise and sink curves. The gradients of these curves give the rise and sink velocities and judging from the non-linear nature of the curves, these velocities are not uniform but change with the intruder position in the bed over time. Nahmad-Molinari et al. [6] have also observed a non-linearity in the rise curve of a 6.32 mm diameter steel ball rising in a granular bed consisting of 2 mm diameter monodisperse cabbage seeds at a frequency of 7.5 Hz with λ varying from 11.5 to 15 mm.

The underlying mechanisms behind the BNE and RBNE have been the subject of disagreement among a number of researchers. In the case of the BNE, the void filling mechanism [10,16], in which the rise of large particles is explained as being due to the falling of smaller particles into voids created underneath the large particles, is one such mechanism. However, a number of researchers [12,14] suggest that air-driven effects play a very significant role in the ascent of large particles in granular beds consisting of fine particles (such as those used in this study). Based on the high-speed movies made, the findings obtained in this study support the explanation that the void filling mechanism is responsible for the ascent of an intruder in a granular bed. These high-speed movies are available for viewing on our website [15]. Given that the intruders used in this study were much heavier and denser than the background particles, convective flows would have had little or no effect on intruder motion. A sequence of movie frames that captures this mechanism is shown in Fig. 7. The void filling mechanism is best appreciated by viewing the movies available online [15]. In essence, it can be explained as follows: due to the force of inertia, the intruder keeps moving upward after the bed has been displaced to the maximum amplitude. This leads to the creation of a low-pressure wake behind the intruder. The wake is then filled with background particles. When the intruder eventually falls down due to gravity, it experiences a net upward displacement.

Fig. 8 shows the trajectory traced out by intruder A7 in the granular bed at a vibration frequency of 22.3 Hz and an amplitude of 2.5 mm. Under these conditions, the intruder rises in the column experiencing the BNE. Observe how the intruder gradually rises with time through a series of up and down displacements in the bed, which trace out parabolic paths. The sinusoidal oscillation of the vibrating bottom is also shown.

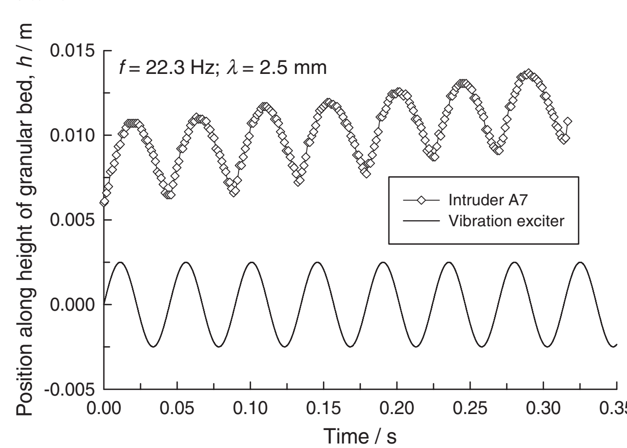

Fig. 8. Parabolic trajectories traced out by intruder A7 as it rises in the granular bed at a vibration frequency f=22.3 Hz and an amplitude λ=2.5 mm. The sine wave oscillation of the vibrating container (bottom) is also shown. Note that the parabolic trajectory of the intruder refers to the bottom of the intruder. Also observe how the intruder gradually rises along the granular bed.

4. Conclusions

The movement of brass intruders in a pseudo-2D granular bed consisting of 0.85 mm diameter polystyrene beads was studied. Based on the work done and results obtained, the following conclusions are hereby reached:

1. A large intruder can either rise or sink in a vertically shaken granular bed depending on the amplitude and frequency of vibration.
2. At a fixed vibration amplitude, the frequency required to cause an intruder to rise is always higher than that required to let it sink.
3. In addition to the vibration amplitude and frequency, the position of an intruder in a granular bed also affects its rise or sink behaviour as shown in Fig. 5(a)–(c). That is, the starting position of an intruder in the bed could also determine whether the intruder rises or sinks.
4. The rise and sink velocity of an intruder is a function of its position in the granular bed and the diameter of the intruder as demonstrated in Fig. 6(a) and (b). Furthermore, at a fixed amplitude, the mean rise velocity of an intruder is always much higher than the mean sink velocity.
5. High-speed video movies support the void-filling mechanism described in the literature [10,16] as the underlying phenomenon responsible for the rise of large particles in a vertically vibrated granular bed.

Nomenclature

d	Inner diameter of intruder, m
D	Outer diameter of intruder, m
f	Vibration frequency, Hz
f_{rise}	Minimum vibration frequency at which an intruder rises from the bottom to the top of the granular bed, Hz
f_{sink}	Minimum vibration frequency at which an intruder sinks from the top to the bottom of the granular bed, Hz
g	gravitational acceleration, m/s^2
m	intruder mass, kg
V	intruder volume, m^3
h	Intruder position from the bottom of the granular bed, m
Γ	Vibration intensity, dimensionless
λ	Vibration amplitude, m
ρ	Density of intruder, kg/m^3
ρ_{b}	Density of granular bed particles, kg/m^3

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