2D Slurry Bubble Column Hydrodynamic Phenomena Clarified with a 3D Gas–Liquid Model

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The gas hold-up in a 2D bubble column is modelled using a 3D gas hold-up model. The influence of the scale of 2D bubble columns on several parameters, for instance, transition gas hold-up, transition gas velocity, and bubble rise velocities, is investigated and related to 3D bubble columns. By adapting the rise velocity of the large bubbles of an existing 3D bubble column model (Krishna et al., 2001a), the gas hold-up in both the homogeneous and the heterogeneous regime can be described satisfactorily. By adjusting the transition points only, it is also possible to describe the gas hold-up in systems containing small amounts of carbon particles and electrolyte. The smallest dimension of the 2D slurry bubble column, the column thickness, influences the location of the regime transition point. In the heterogeneous regime, however, it is only the largest column dimension, the column width, that influences the gas hold-up. These observations together enable proper 2D/3D bubble column comparison in future studies.

Dans cette étude, la rétention de gaz dans une colonne à bulles en 2D est modélisée à l'aide d'un modèle de rétention de gaz en 3D. L'influence de l'échelle des colonnes à bulles 2D sur plusieurs paramètres, comme la rétention de gaz de transition, la vitesse de gaz de transition et les vitesses de montée des bulles, est étudiée et reliée aux colonnes à bulles 3D. On montre qu'en adaptant la vitesse de montée des bulles larges fournie par un modèle de colonnes à bulles 3D existant (Krishna et al., 2001a), la rétention de gaz tant en régime homogène qu'hétérogène peut être décrite de manière satisfaisante. En ajustant seulement les points de transition, il est également possible de décrire la rétention de gaz dans des systèmes contenant de petites quantités de particules de carbone et d'électrolyte. On a trouvé que la plus petite dimension de la colonne à bulles à suspensions 2D, soit l'épaisseur de la colonne, influence la position du point de transition de régime. Cependant, dans le régime hétérogène, c'est seulement la plus grande dimension de la colonne, soit la largeur de la colonne, qui influence la rétention de gaz. Toutes ces observations vont permettre des comparaisons adéquates des colonnes 2D et 3D dans les prochaines études.

Keywords: bubble column, gas hold-up, 2D/3D comparison, modeling, scale.

Modeling of gas hold-up as a function of operating parameters in two and three phase systems is an important subject in multiphase reactor engineering (Krishna et al., 2001a; Bauer and Ellenberger, 2001; Camarasa et al., 2001; Khinast, 2001; Maretto and Krishna, 1999; Meikap et al., 2002). Many correlations have been proposed to describe the gas hold-up in two and three phase systems to predict the effect of addition of surfactants (Zahradnik et al., 1999; Zhang et al., 2001), electrolyte (Kellerman et al., 1998; Zahradnik et al., 1995), and several other substances. However, a fundamental understanding of the physical and hydrodynamic parameters and mechanisms determining the gas hold-up in these multiphase systems is still lacking. Generally, the application of empirical, engineering correlations to systems outside the experimental window in which they were determined, is questionable. Furthermore, model assumptions are sometimes difficult to verify because of restrictions on measuring techniques.

A transparent 2D bubble column can increase the insight into multiphase hydrodynamics, by enabling video recording and image analysis of flow patterns, bubble size distributions, bubble rise velocities, and many other flow characteristics. This insight is increased if the gas hold-up and the effect of the scale of the bubble column can be modelled and clarified with a comprehensive model. This study aims to quantify the gas hold-up in a 2D bubble column, based on a 3D model, and to evaluate the effect of the scale of a bubble column on several hydrodynamic parameters.

The gas hold-up in a bubble column is determined by many parameters, for instance the physical properties of the gas, the liquid, and the solids, the rates of bubble coalescence and break-up, the position of the transition point, the rise velocity of the bubbles in the homogeneous and heterogeneous regimes, and the column scale. The last is one of the most important parameters, because it affects many of the others. To understand the influence of the scale of a bubble column on gas hold-up, it is necessary to understand to what extent these parameters are affected by the column scale. Many of the engineering correlations for the prediction of gas hold-up do not predict the effect of the scale of the column, because these correlations are usually based on dimensionless groups which are fitted on experimental data obtained in one column only. However, in recent years, Krishna et al. have developed a model consisting of separate correlations for the rise velocity of the gas bubbles,

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the gas hold-up and the superficial gas velocity at the transition point, including the effect of bubble-bubble interactions and the effect of bubble-column wall interactions. This model was reported in many articles (Krishna et al., 2001a; Krishna et al., 2001b; Krishna et al., 2000; Krishna et al., 1999; Krishna et al., 1998) and was comprehensively published in the thesis of Urseanu (2000); further reference to this model will be made as Krishna et al. (1999).

The model of Krishna et al. (1999) includes most of the above mentioned factors, influencing the gas hold-up in a 3D bubble column, and is therefore currently the most reliable model for the gas hold-up prediction in a 3D system. Therefore, this model is taken as a starting point to investigate the effect of the scale of a bubble column on model parameters like the bubble rise velocity, including bubble-bubble interactions and bubble-wall interactions, and to describe the gas hold-up in a 2D bubble column. First the experimental setup is introduced in which the gas hold-up measurements were performed. Secondly, the 3D model of Krishna et al. (1999) is treated comprehensively. Hereafter all model parameters are evaluated more closely to determine to what extent these are affected by the scale of a 2D column (viz., column width and column thickness). The model parameters that are clearly most affected are adapted accordingly. Finally, the resulting model with the modified parameters is verified experimentally by comparison with measured gas hold-up data in the 2D bubble column in three different systems.

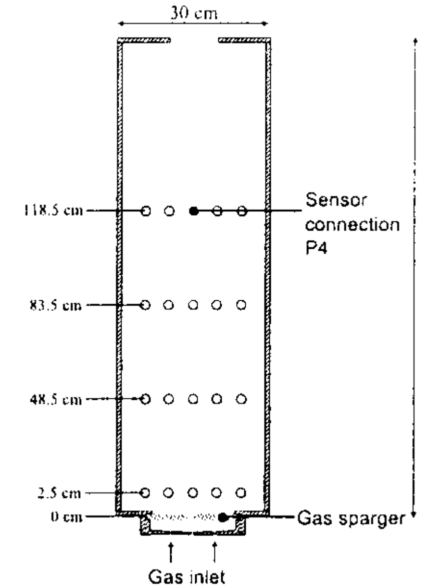


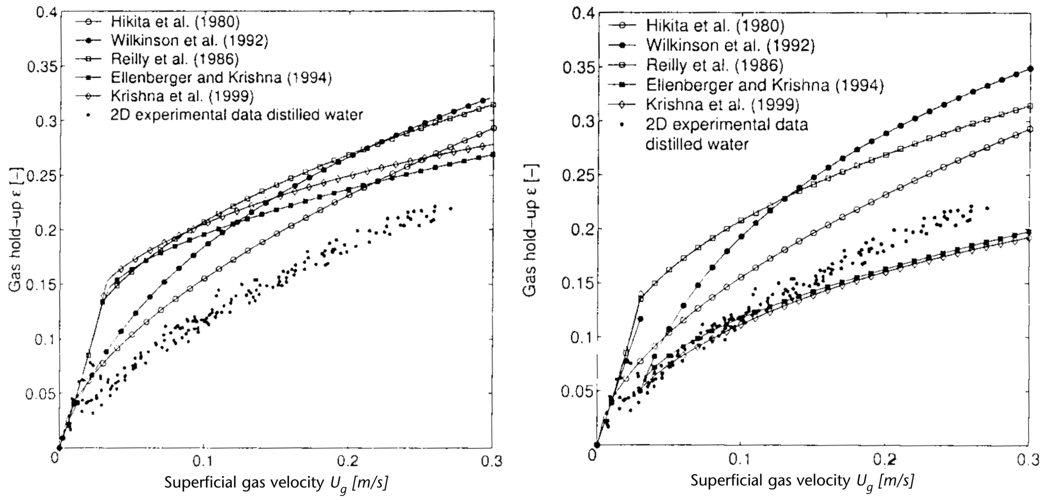
Figure 1. 2D perspex bubble column dxwxh 0.015x0.30x2.00 m, with 20 sensor connections located at 2.5, 48.5, 83.5, and 118.5 cm above the gas sparger, for gas hold-up measurements. Two gas spargers were used: a 0.5 mm perforated plate and a 30 μm porous plate. If not stated otherwise, the 0.5 mm perforated plate was used in the experiments.

2D Experimental Set-up

A 2D bubble column is used to study the bubble flow pattern, bubble size distribution, and bubble rise velocity during gas hold-up measurements, with a high-speed video camera. The 2D laboratory scale reactor shown in Figure 1 consists of two perspex plates with a height of 2 m and a width of 0.3 m. The two walls of the column are placed 0.015 m apart from each other. Gas hold-up measurements are performed under ambient conditions (1 bar, 293 K), with distilled water, with small amounts of catalyst particles (carbon particles, $\overline{dp} \approx 30$ μm, 0.1 – 20 g/l) and with electrolyte (sodium gluconate, 0.05M – 2.0 M). From several experiments it is observed that a small range of liquid viscosity (1.0 to 2.0 kg/m·s), gas density (0.17 to 1.3 kg/m^3), and type of gas (nitrogen, oxygen, and air) do not influence the gas hold-up significantly. All experiments in this paper are therefore carried out with nitrogen gas. Initial liquid height does not influence the gas hold-up if it is kept above 1 m. Therefore in all experiments, the initial liquid height is between 1.0 and 1.5 m. Distilled water is preferred over tap water because the properties of tap water are poorly defined. An extensive flushing procedure is applied to minimize the amount of impurities in the distilled water. Local and overall gas hold-up are calculated from pressure sensors which are connected at the back wall of the 2D column at various positions shown in Figure 1. If the sensors are not used, the sensor connections are closed flush with the wall to prevent disturbances in the flow behaviour. The regime transition point is determined from the dynamic pressure signal. Changes in the average cycle frequency of the pressure signal recorded at a frequency of 25 Hz, are a measure of the transition from the homogeneous regime to the heterogeneous regime (Kluytmans et al., 2001). Video images are recorded with a high speed Dalsa CA-D6 camera at a frequency of 955 frames per second. The video images are analyzed with image processing software developed at the Eindhoven University of Technology, to obtain quantitative data about bubble size distributions and bubble rise velocities. These image processing software can be found at our Web site (http://www.chem.tue.nl/scr).

3D Gas Hold-up Correlations and Models

Many models and empirical correlations are available to predict the gas hold-up in two and three phase bubble columns. The models and correlations of Hikita et al. (1980), Reilly et al. (1986), Wilkinson et al. (1992), Ellenberger and Krishna (1994), and Krishna et al. (1999) are compared with the experimental data obtained in the 2D bubble column (Section 2). The experimental conditions for which these correlations were developed mostly resemble the experimental conditions of those in the 2D gas hold-up measurements of Kluytmans et al. (2001). Figure 2a shows the gas hold-up in the homogeneous regime, below a superficial gas velocity of 0.015 m/s, and is reasonably well described by the selected models and correlations. However, the gas hold-up in the heterogeneous regime is over-predicted by all models, although the shape and slope of some curves resemble the shape and slope of the measured data quite well. Besides the deviation between the predicted and measured gas hold-up, a large difference exists between the calculated transition points from the 3D models and the measured transition point in the 2D bubble column, as shown in Table 1. This deviation might be caused by a difference in the definition of the transition point within the transition regime. The transition point in this paper will be taken at the start of the transition regime. The validity of this definition is addressed in Section 6. The gas hold-up correlations and models based on the transition points mentioned in Table 1 then result in the curves shown in Figure 2b. This comparison shows that the correlation of Hikita et al. (1980), and the models of Ellenberger and Krishna (1994) and Krishna et al. (1999) are the most promising for predicting the gas hold-up in the 2D slurry bubble



(a) "Using calculated transition points" (b) "Using measured transition point"

Figure 2. Comparison of literature models and correlations with experimental data of gas hold-up obtained in a 2D bubble column. a) Transition points as calculated by the models and correlations, b) Experimentally determined transition point in the 2D bubble column, see Table 1.

Table 1. Calculated and measured transition parameters for the transition from the homogeneous to the heterogeneous regime. Systems: distilled water - nitrogen or distilled water - air. Experimental study: 2D bubble column; literature values: 3D bubble columns.

	ε_{trans} [-]	U_{trans} [m/s]
Wilkinson et al. (1992)	0.01	0.0026
Reilly et al. (1986)	0.1342	0.0296
Krishna et al. (1999)	0.1517	0.0305
Experimental study (Kluytmans et al., 2001)	0.05 ± 0.01	0.03 ± 0.01

column. The correlation of Hikita et al. (1980) is, however, purely empirical and does not offer much possibility to explore the effect of the scale of the 2D column on separate model parameters, such as bubble rise velocity and transition points. Therefore this correlation is not further considered. The Krishna et al. (Krishna et al., 1999; Krishna et al., 2001a; Urseanu, 2000; Ellenberger and Krishna, 1994) model is built on fundamental and semi-empirical correlations describing separate parameters, like the rise velocity of a single bubble, the rise velocity of a bubble swarm, the transition point, etc. By adapting these sub-correlations to t the 2D gas hold-up data, insight can be obtained to what extent the separate model parameters are affected by the scale of the 2D bubble column compared to the 3D case.

Gas Hold-up Model by Krishna et al. (1999)

Ellenberger and Krishna (1994) studied the analogy between gas–solid fluidized beds and gas–liquid-solid bubble column reactors. Their model is based on the two-phase model for gas–solid systems of May (1959) and van Deemter (1961), in which the gas phase is divided into a large bubble phase and a dense phase containing only small bubbles. This distinction was made to account for the different behaviour of small and large bubbles in multiphase reactors. Based on these considerations, Ellenberger and Krishna (1994) developed a model for the prediction of the gas hold-up in the heterogeneous regime in gas-liquid bubble columns. Krishna et al. (1999) extended this model with a correlation for the gas hold-up in the homogeneous regime. The model of Krishna et al. (1999) consists of separate sets of equations for the homogeneous and the heterogeneous regime. The model parameters are treated in the next sections, and evaluated on their potential to be affected by the column dimensions. This evaluation leads to the insight about which model parameters need to be adapted based on the 2D/3D scale difference.

Homogeneous Regime

In the homogeneous regime it is assumed that only equally sized gas bubbles are present. It is assumed that these small gas bubbles rise with the same and constant velocity throughout the column. Once the rise velocity of these small bubbles has been estimated, the gas hold-up can be calculated with:

$$\varepsilon_g = \frac{U_g}{U^{\infty}_{small,b}\left(1-\varepsilon_g\right)} \quad \text{for} \quad U_g < U_{trans} \qquad (1)$$

The definition of the symbols can be found in Section 9. The factor $U^{\infty}_{small,b}(1-\varepsilon_g)$ in Equation (1) accounts for the fact that the rise velocity of the small bubbles is retarded by the presence of surrounding bubbles as was described by Darton and Harrison (1975). The average diameter of the small gas bubbles in the homogeneous regime is considered to be between 4 and 8 mm (Krishna et al., 2001a). Therefore it is expected that if the 2D column thickness exceeds 1 cm, it will hardly affect the rise velocity of the small bubbles, and therefore will not affect the gas hold-up in the homogeneous regime.

Transition Regime

The transition region separating the homogeneous regime and the fully developed heterogeneous regime is not considered in the model of Krishna et al. (1999). In the model of Krishna et al. (1999) this region is reduced to a transition point, which is located at the intersection of the gas hold-up correlation for the

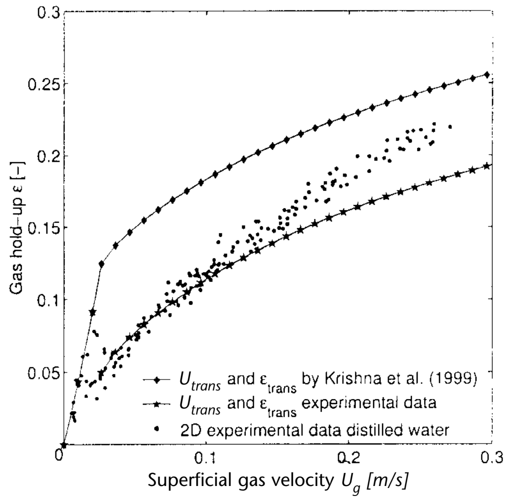


Figure 3. Gas hold-up prediction with the model of Krishna et al. (1999) with the measured transition point in a 2D bubble column (Kluytmans et al., 2001) and with the calculated transition parameters, compared with experimental hold-up data measured in the 2D laboratory column.

homogeneous regime and the gas hold-up correlation for the heterogeneous regime. Generally, it is found that the transition regime lies between superficial gas velocities of 0.05 m/s and 0.15 m/s. At low superficial gas velocities in the transition regime, the first large bubbles are formed due to mutual interactions of the small bubbles while at higher superficial gas velocities larger bubbles start to interact. The location of the transition point thus strongly depends on its definition as well as on the measuring technique or the calculation procedure.

The transition point is one of the critical parameters in the model of Krishna et al. (1999),because it determines the end point of the correlation describing the gas hold-up in the homogeneous regime and the starting point of the correlation for the heterogeneous regime. Using an experimentally determined transition point can lead to a discontinuity in the prediction of the gas hold-up, as shown in Figure 3, in which both the experimentally determined transition point and the calculated transition point are used. Therefore, Krishna et al. (1999) proposed an empirical correlation for the transition hold-up using dimensional analysis:

$$\varepsilon_{trans} = 0.012 Re_b^{0.4} We^{-0.2}\left(1-\exp\left(-0.04\frac{D_T}{d_b}\right)\right) \tag{2}$$

Here, the Reynolds number is based on the average bubble diameter d_b at the transitionpoint, and is defined as $Re_b = (\rho_{liquid}\overline{U}_b d_b)/\eta_{liquid}$, while the Weber number is described as $We = (g d_b^2 \rho_{liquid})/\sigma$. The transition gas velocity is defined by Reilly et al. (1986):

$$U_{trans} = U^{\infty}_{small,b}\varepsilon_{trans}(1-\varepsilon_{trans}) \tag{3}$$

which is equal to Equation (1) employed at the upper boundary of the dispersed bubble flow. Equations (2) and (3) were found to predict the transition gas hold-up and superficial gas velocity in air-water systems and air-tellus oil systems in several 3D columns. It is expected that the scale of the column will have a large influence on both the superficial gas velocity and the gas hold-up at the transition point. For example, interactions of the bubbles with the column 5 walls are assumed to promote bubble coalescence. This consideration is expressed in Equation (2) in which both the bubble diameter and the column diameter are included. However, for a 2D system it is unknown which column dimension should be used in this equation in order to calculate the correct gas hold-up at the transition point.

Heterogeneous Regime

The gas hold-up in the heterogeneous regime is obtained by addition of the gas hold-up in the homogeneous regime, the dense phase gas hold-up, and the gas hold-up of the large bubbles. It is assumed that the gas hold-up of the small bubbles (dense phase) is equal to the gas hold-up at the transition point, and is constant throughout the heterogeneous regime. The gas hold-up for the heterogeneous regime is then given as:

$$\varepsilon_g = \varepsilon_{l,b} + \varepsilon_{trans}(1-\varepsilon_{l,b}) \quad \text{for} \quad U_g > U_{trans} \tag{4}$$

By definition, the gas hold-up of the large bubbles is given by,

$$\varepsilon_{l,b} = \frac{U_g - U_{trans}}{U_{l,b}} \tag{5}$$

The velocity of the large bubbles $U_{l,b}$ is related to the rise velocity of a single bubble in an infinite medium as given by Davies and Taylor (1950):

$$U^{\infty}_{l,b} = \Phi\sqrt{g d_b} \tag{6}$$

with $\Phi = 0.71$. However, in a bubble column, bubbles have mutual interactions as well as interactions with the column walls. The rise velocity of a single bubble, interacting with other gas bubbles and with the column walls, is therefore expressed as:

$$U_{l,b} = 0.71\sqrt{g d_b}(SF)(AF) \tag{7}$$

SF is the so called scale factor and *AF* is the acceleration factor. The scale factor *SF* was introduced by Collins (1967) to account for the bubble-wall interactions. The scale factor is given by an empirical correlation and is a function of the ratio between the bubble diameter and the diameter of the column:

$$SF = 1 \qquad \text{for } \frac{d_b}{D_T} < 0.125$$

$$SF = 1.13 exp\left(-\frac{d_b}{D_T}\right) \qquad \text{for } 0.125 < \frac{d_b}{D_T} < 0.6 \tag{8}$$

$$SF = 0.496\sqrt{\frac{dT}{D_b}} \qquad \text{for } \frac{d_b}{D_T} > 0.6$$

The mutual interactions of the gas bubbles are accounted for by Krishna et al. (1999) through the acceleration factor (*AF*). This empirical parameter was fitted for low viscosity fluids, resulting in the following correlation:

$$AF = 2.73 + 4.505\left(U_g - U_{trans}\right) \tag{9}$$

Combining Equations (7) and (5) provides the prediction of the large bubble gas hold-up in the heterogeneous regime:

$$\varepsilon_{l,b} = \frac{U_g - U_{trans}}{0.71\sqrt{g d_b}(SF)(AF)} \tag{10}$$

From the above equations it can be seen that the gas hold-up in the heterogeneous regime is mainly determined by the gas velocity at the transition point and the rise velocity of the large gas bubbles in the column. It is not known to what extent the rise velocity of the large bubbles and the bubble-bubble interactions expressed by the acceleration factor are affected by the column diameter. Furthermore, the applicability of the scale factor correlation of Collins (1967) for 2D bubble columns requires further investigation.

2D Modeling

As raised in the previous section, the basic question in the modeling of the gas hold-up in a 2D bubble column with the 3D model by Krishna et al. (1999), is which characteristic column size (viz., column width or column thickness) should be used in the calculation of the transition point, the scale factor, and the rise velocity of the large bubbles. To answer this question, first a model sensitivity analysis is performed to determine which parameters influence the gas hold-up most. This is done by comparing gas hold-up model predictions with the gas hold-up data that were measured in the 2D laboratory scale column (Section 2). Subsequently, the influence of column size on the model parameters that affect the hold-up most is determined and these model parameters are adapted accordingly.

Sensitivity Analysis

Four parameters are selected which are assumed to be most affected by the scale of the column, viz. the transition superficial gas velocity (U_{trans}), the transition gas hold-up (ε_{trans}), the scale factor (*SF*), and the acceleration factor (*AF*). The sensitivities of the gas hold-up prediction to changes in these parameters are shown in Figures 4a to 4d. These figures show that the effect of the transition parameters and the acceleration factor on the gas hold-up prediction is relatively limited. The scale factor however, influences the gas hold-up prediction to a large extent.

Homogeneous Regime

The gas hold-up in the homogeneous regime is governed by the rise velocity of the small bubbles, which is determined in the experimental studies of Krishna et al. (1999), Reilly et al. (1986), and Wilkinson et al. (1992). The rise velocity of the small bubbles in the 2D bubble column is measured with high speed video imaging. The results are compared with the literature values in Table 2. It is clear that the rise velocity of the small bubbles and thus the gas hold-up is not affected by the column size.

Heterogeneous Regime

Equations (5) and (7) show that the rise velocity of the large bubbles, influenced by the scale factor and the acceleration factor, determine the gas hold-up in the heterogeneous regime. A sensitivity analysis shows that the acceleration factor has only a minor influence on the gas hold-up prediction while the scale factor has a much larger influence. These parameters will be treated separately.

Acceleration Factor

The acceleration factor accounts for the effect of the mutual interactions of the bubbles on the rise velocity of the bubbles. The acceleration factor depends on the superficial gas velocity, because the contribution of the large and small bubbles to the gas hold-up changes with increasing gas velocity. The parameters in the *AF* correlation (Equation 9) were fitted by Krishna et al. (1999) with 3D experimental data. Figure 4c shows that this correlation describes the measured 2D gas hold-up data reasonably well. This supports the idea that the mutual bubble interactions, which the acceleration factor accounts for, are not affected by the size of the column. Therefore, Equation (9) can be used for the prediction of the gas hold-up in a 2D bubble column.

Scale factor

The scale factor (*SF*) introduced by Collins (1967) has been derived explicitly for 3D bubble columns. This scale factor adapts the factor $\Phi = 0.71$ in the theoretical equation of Davies and Taylor (1950) for the rise velocity of a single bubble in an infinite medium, to account for the effect of the column size. Krishna et al. (2000) have derived a scale factor for 2D columns, based on experiments and CFD modelling on the rise velocity of single bubbles, in a 2D column with a column thickness of 5 mm. Using this 2D scale factor for the estimation of the gas hold-up in the 2D column in the present work, does not give a satisfactory description of the measured gas hold-up data. This is possibly due to the difference in the thickness of the 2D columns that were used in both studies, viz. 5 mm in the study of Krishna et al. (2000) and 15 mm in the 2D column in the present work. The mutual interactions between the small and large bubbles will most probably be influenced by this distance between the column walls. In the work of Krishna et al. (2000), the column thickness is in the order of magnitude of the size of the small bubbles (4 to 8 mm), while in the 2D setup in the present work, the column thickness is at least twice as large as the small bubble size. Therefore, in this case, the rise velocity of the large bubbles has to be adapted differently to account for the effect of the column size than by the 2D scale factor

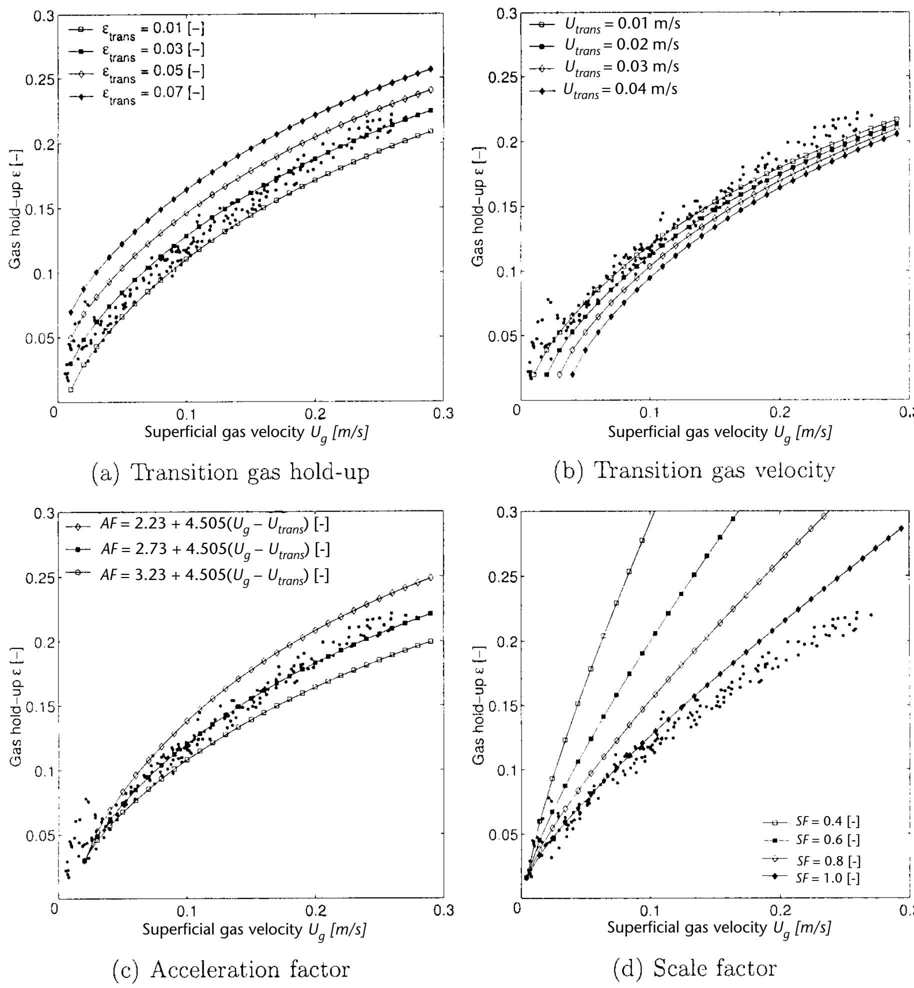


Figure 4. Evaluation of model parameters of the 3D model of Krishna et al. (1999) with respect to experimental data obtained in a 2D bubble column. Model parameters were changed to evaluate the sensitivity of the gas hold-up prediction with respect to these parameters. A) the transition hold-up, b) the superficial gas velocity at the transition point, c) the acceleration factor, and d) the scale factor. System: nitrogen distilled water.

Table 2. Literature values of the rise velocities of small bubbles in the homogeneous regime (3D) and measured rise velocity of small bubbles in the 2D bubble column (ambient conditions; system: distilled water-air).

	$U_{small,b}$ [m/s]
Wilkinson et al. (1992)	0.26
Reilly et al. (1986)	0.24-0.26
Krishna et al. (1999)	0.23-0.25
This study (2D column)	0.25 ± 0.02

introduced by Krishna et al. (2000). Pyle and Harrison (1967) have adapted the factor Φ = 0.71 for a 2D column with a thickness of 1 cm, to Φ = 0.54. The latter value adapts the rise velocity of a single bubble under the influence of the smallest dimension of the 2D column, viz. the thickness of the column. In their case, the 2D column thickness is also larger than the size of the small bubbles. Applying this to the correlation of Krishna et al. (1999) results in a rise velocity of the large bubbles of:

$$U_{l,b} = 0.54\sqrt{gd_b}(SF)(AF) \tag{11}$$

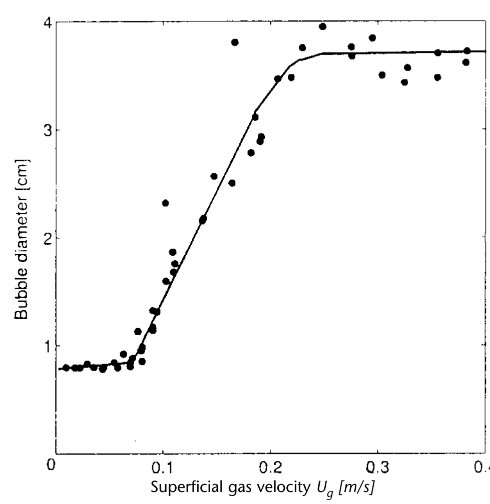


Figure 5. Average bubble size as a function of the superficial gas velocity, calculated from video images captured during gas hold-up experiments at a frame rate of 955 Hz. System: Nitrogen-Carbon particle slurries 0.1 to 2.0 g/L.

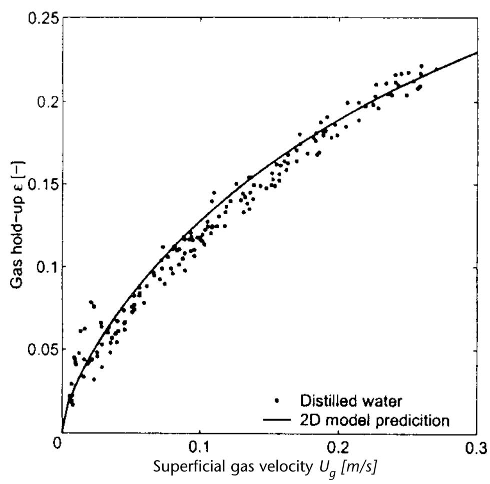
Table 3. Values and equations of the model parameters for the modeling of the gas hold-up in a 2D bubble column.

Parameter	Value/Equation
$U_{small,b}$	0.25 [m/s]
AF	$2.73 + 4.505(U_g - U_{trans})$ [-]
SF	1 [-]
$U_{l,b}$	$0.54\sqrt{g d_b}\,(SF)(AF)$ [m/s]
d_b	Values according to Figure 5 [m]

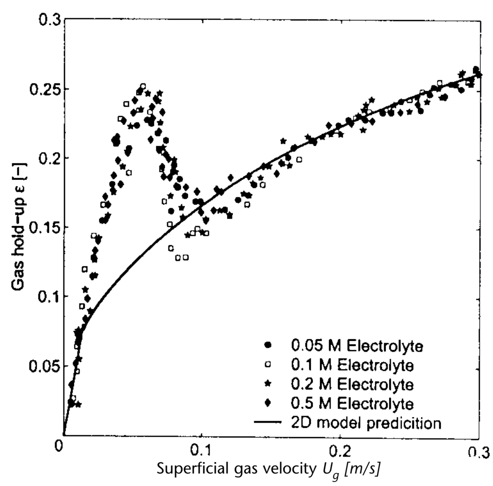
The scale factor of Collins (1967) should in this case only correct for the influence of the width of the 2D column on the rise velocity of the large bubbles. To calculate the value of the scale factor according to Equation (8), the average bubble size of the large bubbles in the heterogeneous regime is required. This average bubble size is estimated from video images recorded with a high speed video camera. Figure 5 shows that the average large-bubble diameter never exceeds the size of approximately 4 cm for gas velocities up to 0.4 m/s. Therefore, the ratio d_b/D_T in Equation (8) is always smaller than 0.125 for a column diameter taken as the column width of the 2D column of 0.3 m; the scale factor in that case is equal to 1. This is in agreement with Figure 4d which shows that scale factors smaller than 1 do not describe the experimental data satisfactorily. This analysis shows that indeed the rise velocity of the large bubbles in the heterogeneous regime is affected by the size of the 2D bubble column.

2D Gas Hold-up Model Validation

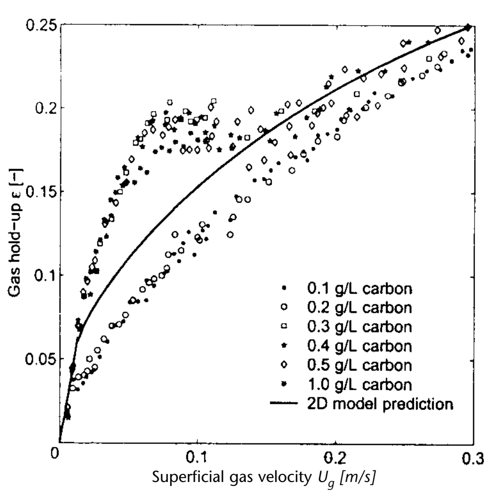
The values of the model parameters to calculate the gas hold-up in a 2D column are summarized in Table 3. The gas hold-up model predictions are compared with the experimental data obtained in the 2D bubble column described in Section 2. For the model predictions, the transition points are so chosen that the gas hold-ups in the three systems in both the homogeneous regime and the heterogeneous regime ($U_g > 0.15$ m/s) are well described. The gas hold-up prediction for carbon particles is optimized for the carbon particle concentrations above 0.3 g/l because the gas hold-up at lower concentrations is equal to the gas hold-up of distilled water, as shown in Figures 6a and 6c.



(a) Distilled water



(b) Electrolyte solutions



(c) Carbon slurries

Figure 6. Measured gas hold-up data in a 2D slurry bubble column modelled with the adapted 3D gas hold-up model of Krishna et al. (1999) for a) distilled water, b) electrolyte solutions, and c) carbon slurries. Data was modelled with $SF = 1$, $AF = 2.73 + 4.505(U_g - U_{trans})$, and the Pyle an Harrison (1967) factor of 0.54 for the rise velocity of the large bubbles, see Table 3.

Table 4. Transition points determined from the optimized gas hold-up prediction in the homogeneous regime and the heterogeneous regime as shown in Figures 6a to c.

System	ε_{trans} [-]	U_{trans} [m/s]
Distilled water	0.017	0.005
Electrolyte solution	0.075	0.014
Carbon slurry	0.06	0.013

The predictions of the gas hold-up for the three experimental systems are shown in Figures 6a - 6c. For the three systems, the gas hold-up is well described in both the homogeneous and the heterogeneous regime. It can be seen that the gas hold-ups in the transition regime for electrolyte solutions (Figure 6b) and carbon particle slurries (Figure 6c) between superficial gas velocities of 0.02 and 0.12 m/s are not predicted at all. These transition regimes are reduced in the predictions to a transition point. The determined transition points are listed in Table 4. The transition hold-up deviates quite extensively from the transition hold-up predicted with Equation (2), when the width of the 2D column (0.3 m) is used as column diameter D_T . Equation (2) gives a transition gas hold-up of $\varepsilon_{trans} = 0.10 – 0.15$, which is at least twice the gas hold-ups given in Table 4. This demonstrates that the transition point is influenced by the size of the column. It is expected that the formation of the first large bubbles at the start of the transition regime is influenced by the smallest dimension of the column. This idea is confirmed when calculating the column diameter D_T from Equation (2) using the transition gas hold-up as given in Table 4. This calculation results in a column diameter for the three transition points (viz., distilled water, electrolyte solution, and carbon slurry) between 3 and 5 cm, which is of the order of magnitude of the smallest dimension of the 2D column (viz., the column thickness of 1.5 cm). Possibly, the thickness of the 2D column influences the development of the first large bubbles by pushing the small bubbles together, therefore forcing the small bubbles to interact and coalesce to form large bubbles. The width of the 2D column has evidently no pronounced effect on this process.

2D - 3D Gas Hold-up Comparison

In the previous sections, we have shown that the gas hold-up in a 2D bubble column is predicted quite well by the model of Krishna et al. (1999) after adapting the calculation of the transition point and the rise velocity of the large bubbles to account for the proper scale of the column (viz., column width or column thickness). The way these parameters had to be adapted allowed insight into the hydrodynamic behaviour of a 2D bubble column. Using both models, we can now make a proper comparison between the gas hold-up in a 2D bubble column and the gas hold-up in a 3D bubble column, at the same superficial gas velocity. This comparison is shown in Figure 7a. It is evident that there is quite a difference in the gas hold-up in the 2D and 3D cases up to a superficial gas velocity of approximately 0.2 m/s, above which the gas hold-up predictions become quite similar. At higher gas velocities, the 2D and 3D gas hold-up predictions remain still very close (see Figure 7b). This would suggest that hold-up data measured at sufficiently high superficial gas velocity in a 2D bubble column, offer a reasonable prediction of the hold-up in the actual 3D case. However, it is expected that this will not be true for any 2D bubble column. The choice of the thickness of the 2D column is crucial in this respect, to properly account for the presence of small bubbles that are approximately 4 to 8 mm in size. A column thickness of at least 1 cm, but preferably 1.5 cm, will allow the small bubbles to flow freely within the liquid, without being pushed upward due to interactions with the column walls. This does justice to the considerations of the two fluid model. It is in this light that the rise velocity of the small bubbles in the homogeneous regime is not affected by the thickness of the column. However, it is expected that for 2D columns with a thickness of less than 1 cm, that this may not be true anymore. In that case, the analysis of the model parameters needs further modification.

Conclusions

The 3D model of Krishna et al. (1999) can be adapted quite easily to properly describe the gas hold-up in a 2D bubble column. The following considerations and model adaptations have been discussed (see also Table 3):

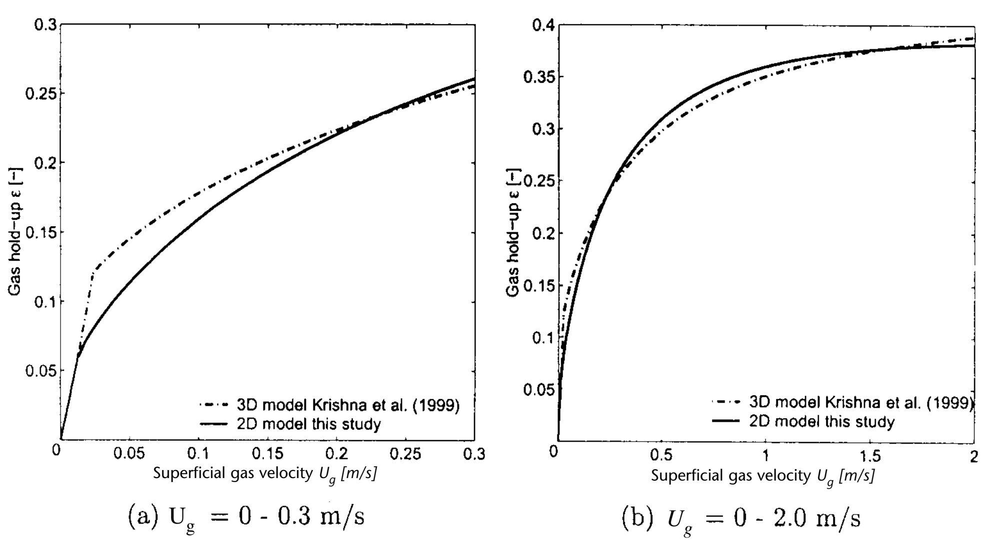


Figure 7. Comparison of the model of Krishna et al. (1999) and the adapted model for a 2D column at a) superficial gas velocities up to 0.3 m/s and b) superficial gas velocities up to 2.0 m/s.

- The 2D column should have a thickness of at least 1 cm but preferably 1.5 cm, to prevent the rise velocity of the small bubbles being affected by the column walls. Generally, the ratio between the 2D column thickness and the average diameter of the small bubbles should exceed the value of 1.
- The rise velocity of the small bubbles in the homogeneous regime equals 0.25 m/s and is independent of the size of the bubble column.
- The transition gas hold-up is affected by the thickness of the 2D column. This means that if the transition gas hold-up is calculated with Equation (2), the characteristic column size D_T in this equation should be taken equal to the thickness of the column and not equal to the width of the column.
- The rise velocity of the large bubbles should be adapted to account for the width of the 2D column by using the factor Φ =0.54 as was suggested already by Pyle and Harrison (1967).
- The scale factor *SF* in case of a 2D column can be calculated with Equation (8) in which the characteristic column size D_T should be taken equal to the column width of the 2D column.
- The acceleration factor *AF* is not affected by the scale of the column and is similar for the 2D and 3D cases.
- The 2D gas hold-up model can be applied to several different systems (distilled water, carbon slurries with small concentrations of carbon particles, electrolyte solutions) by adjusting the transition points.

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Nomenclature

AF	acceleration factor
d_b	bubble diameter, (m)
g	acceleration constant, (m/s^2)
Re_b	Reynolds number based on bubble diameter, $(\rho_{liquid}\bar{U}_b d_b)/\eta_{liquid})$
U_{trans}	transition gas velocity, (m/s)
$U_{l,b}$	large bubble rise velocity, (m/s)
$U_{l,b}^{\infty}$	large bubble rise velocity in a infinite medium, (m/s)
U_g	superficial gas velocity, (m/s)
$U_{small,b}$	small bubble rise velocity, (m/s)
$U_{small,b}^{\infty}$	small bubble rise velocity in a infinite medium, (m/s)
SF	Scale factor
We	Weber number, $((gd_b^2\rho_{liquid})/\sigma)$

Greek and Roman Symbols

ε_{trans}	transition gas hold-up
$\varepsilon_{l,b}$	large bubble gas hold-up
ε_g	gas hold-up
η_{liquid}	liquid viscosity, (Pa·s)
ρ_{liquid}	liquid density, (kg/m^3)

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