Influence of Increased Gas Density on Hydrodynamics of Bubble-Column Reactors

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A mechanistic background to the understanding of the hydrodynamics of high-pressure bubble column reactors in both the homogeneous and heterogeneous flow regimes is discussed. An important parameter determining the stability of homogeneous bubbly flow in a bubble column is shown to be the Richardson-Zaki exponent in the bubble swarm velocity relationship $V_{swarm} = v_\infty (1-\epsilon)^{n-1}$. Experimental data for the bubble swarm velocity were obtained in 0.05- and 0.1-m-dia. bubble columns with various gases (helium, air, argon, sulfur hexafluoride) using water as the liquid phase. Bubble swarm velocity data show that with increasing gas density the Richardson-Zaki exponent value decreases; physically this means that increasing gas density reduces interaction between neighboring bubbles and, consequently, reduces chance of propagation of instabilities. This rationalizes the experimental observation that the influence of increased gas density ρ_G is to delay the transition from homogeneous bubbly flow to churn-turbulent flow: increasing ρ_G increases the regime transition velocity. A stability analysis rationalizes the observations.

The hydrodynamics of bubble columns in the churn-turbulent regime is considered to be analogous to that of a bubbling gas-solid fluidized bed, and the two-phase theory of gas-solid fluid beds is extended to describing bubble columns by identifying the "dilute" phase as the fast-rising large bubbles and the "dense" phase as the liquid phase with entrained small bubbles. A simple coalescence rationalizes experimental large-bubble holdup data.

Introduction

Bubble column reactors, with or without suspended solids, are widely used in industry for a variety of chemical processes (Deckwer, 1992; Fan, 1989). Industrial gas-liquid bubble column reactors are often operated at high pressures but have become the subject of experimental studies only recently (Clark, 1990; Hikita et al., 1980; Idogawa et al., 1986; Krishna et al., 1991; Oyevaar, 1989; Öztürk et al., 1987; Reilly et al., 1986; Wilkinson, 1991; Wilkinson et al., 1992). These studies show that the influence of increased gas density is to significantly increase the gas holdup. It has been shown by Deckwer (1986) and Shah and Deckwer (1985) that the conversion of a bubble column reactor can decrease significantly on transition from homogeneous to the heterogeneous regime.

In an initial attempt to explain the influence of increased gas density on bubble column hydrodynamics, Krishna et al. (1991, 1993) suggested a simple mechanistic model containing three main facets as discussed below.

(i) Homogeneous bubbly flow regime prevails at superficial gas velocities U below a value U_{trans}. In the homogeneous bubbly flow regime, the bubbles are roughly of uniform size and the gas holdup in this regime is calculated from

$$\epsilon = \frac{U}{V_{small}} \quad \text{(homogeneous flow regime, } U \leq U_{trans}\text{)} \qquad (1)$$

where V_{small} is the rise velocity of the small bubble population.

(ii) At a superficial gas velocity, U, greater than U_{trans}, the heterogeneous flow regime prevails and is characterized by the coexistence of small and large bubbles. The large bubbles rise through the reactor virtually in plug flow while the small bub-

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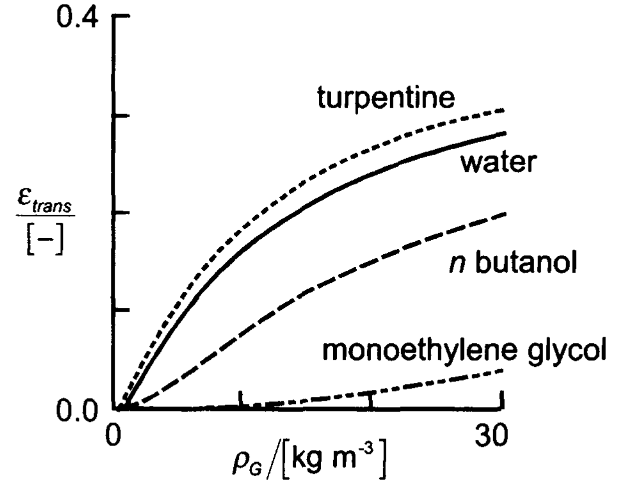

Figure 1. Influence of gas density on superficial gas velocity at the regime transition point, ϵ_{trans}.

Calculations using Eq. 3. Data used for the systems: (i) water: $\rho_L = 1{,}000$; $\sigma = 0.072$; $\mu_L = 0.001$ (ii) monoethylene glycol: $\rho_L = 1{,}113$; $\sigma = 0.048$; $\mu_L = 0.021$; (iii) n butanol: $\rho_L = 812$; $\sigma = 0.0246$; $\mu_L = 0.0034$; (iv) turpentine: $\rho_L = 761$; $\sigma = 0.0247$; $\mu_L = 0.00094$.

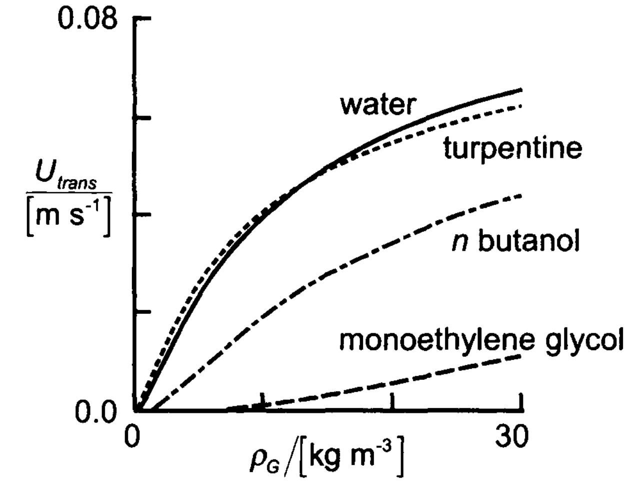

Figure 2. Influence of gas density on superficial gas velocity at the regime transition point, U_{trans}.

Calculations using Eqs. 1-4. Data used are the same as in Figure 1.

bles are "entrained" in the liquid phase and have essentially the backmixing characteristics of the liquid phase. The heterogeneous flow regime has remarkable similarities with a bubbling gas-solid fluidized bed and Krishna (1993) has suggested the use of the two-phase theory of fluid beds for modeling a bubble column reactor in this regime. The total gas holdup in this regime is a sum of two contributions: the small bubble holdup, equal to the total gas holdup at the velocity U_{trans} and the holdup of fast-rising large bubbles, ϵ_{large}:

$$\epsilon = \epsilon_{trans} + \epsilon_{large};\quad \epsilon_{trans} = \frac{U_{trans}}{V_{small}};\quad \epsilon_{large} = \frac{(U - U_{trans})}{V_{large}}$$

(heterogeneous flow regime, $U > U_{trans}$) (2)

where V_{large} is the rise velocity of the large bubble population.

(iii) The influence of increased gas density is to stabilize the homogeneous flow regime and thus delay the transition from homogeneous to heterogeneous, or churn-turbulent, flow regime. Put another way increasing ρ_G has the effect of increasing the value of U_{trans}.

Recently, Wilkinson et al. (1992), using an extensive database and with the Krishna et al. (1991) model as a basis, developed the following correlations for ϵ_{trans}, V_{small} and V_{large}:

$$\epsilon_{trans} = \exp(-193\ \rho_G^{-0.61} \mu_L^{0.5} \sigma^{0.11}) \quad (3)$$

$$\frac{V_{small}\mu_L}{\sigma} = 2.25 \left(\frac{\sigma^3 \rho_L}{g \mu_L^4}\right)^{-0.273} \left(\frac{\rho_L}{\rho_G}\right)^{0.03} \quad (4)$$

$$\frac{V_{large}\mu_L}{\sigma} = \frac{V_{small}\mu_L}{\sigma} + 2.4 \left(\frac{(U - U_{trans})\mu_L}{\sigma}\right)^{0.757} \left(\frac{\sigma^3 \rho_L}{g \mu_L^4}\right)^{-0.077} \left(\frac{\rho_L}{\rho_G}\right)^{0.077} \quad (5)$$

Correlations 3-5 were developed by Wilkinson et al. (1992), not by direct measurements of ϵ_{trans}, V_{small} and V_{large}, but by *interpretation* of total gas holdup data in terms of the model suggested by Krishna et al. (1991) and outlined above (see Eqs. 1 and 2). The important influence of increased gas density on flow regime transition in bubble columns is underlined by calculations for ϵ_{trans} and U_{trans} for four typical liquids: water, turpentine, monoethylene glycol and n butanol as a function of gas density using Eqs. 1-4; the results are shown in Figures 1 and 2. It is seen that both gas density and liquid properties have a significant influence on the values of ϵ_{trans} and U_{trans}. The first major purpose of this article is to employ a linear stability analysis to rationalize the observation that increased gas density delays transition to the homogeneous flow regime.

Calculations for the large bubble holdup ϵ_{large} using Eqs. 2-

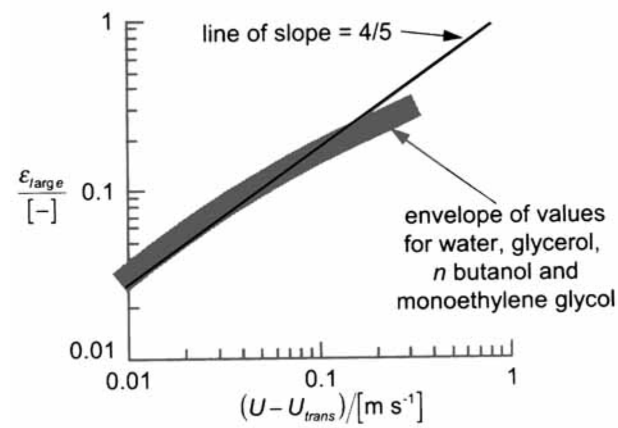

Figure 3. Large-bubble holdup ϵ_{large} as a function of $(U - U_{trans})$ for gas liquid systems.

The thick gray region represents the envelope of calculations using Eqs. 2, 4 and 5 for the systems water, monoethylene glycol, n butanol and turpentine. The gas density used is $\rho_g = 1.5\ \mathrm{kg \cdot m^{-3}}$; other properties used are the same as in Figure 1.

4 shows a remarkably small dependence on liquid-phase properties and appears to be a unique function of the superficial gas velocity through the "dilute" phase ($U - U_{trans}$), see Figure 3. The second major purpose of this article is to develop, relying on the analogy with gas-solid fluidized beds, a simple physical model for the large bubble holdup. We shall demonstrate that a more fundamentally based model suggests correlating the experimental data in the form $\epsilon_{large} \propto (U - U_{trans})^{4/5}$ where the four-fifths power derives from a coalescence model of Darton et al. (1977).

We start by analyzing the stability of homogeneous bubbly flow using the theory of Biesheuvel and Gorissen (1990) as a starting point.

Stability of Homogeneous Bubbly Flows

Biesheuvel and Gorissen (1990) have presented a theoretical analysis of stability of homogeneous bubbly flows subject to void fraction disturbances. They develop the following criterion for *instability* of homogeneous bubbly flow:

$$-\epsilon^2 \frac{\partial v_0}{\partial \epsilon} \frac{\partial}{\partial \epsilon}\left[\left(\rho_G + \frac{1}{2}\rho_L m_0\right) v_0\right] \geq -\left[\frac{\partial p_e}{\partial \epsilon} + \frac{12\pi a \mu_L}{\frac{4}{3}\pi a^3} \frac{1}{(1-\epsilon)^n} \delta_e\right] \quad (6)$$

where we largely retain the nomenclature of Biesheuvel and Gorissen (1990) and take the opportunity of pointing out a sign error in the first right member of their final result Eq. 61. Replacing $\geq$ in Eq. 6 with an equality allows calculation of the maximum, stable, gas holdup ϵ_{trans} for homogeneous bubbly flow. The physical significance of some of the important parameters in Eq. 6 are discussed below.

v_0 is the velocity of the bubble swarm in the zero volume flux frame; this velocity is related to the single, isolated, bubble rise velocity, v_∞, by:

$$v_0 = v_\infty (1-\epsilon)^n \quad (7)$$

when n is the Richardson-Zaki exponent. In the laboratory fixed reference frame, the bubble swarm velocity is (Richardson and Zaki, 1954; Wallis, 1969)

$$V_{swarm} \equiv \frac{U}{\epsilon} = \frac{v_0}{(1-\epsilon)} = v_\infty (1-\epsilon)^{n-1} \quad (8)$$

The stability criterion Eq. 6 can be understood as follows. A displacement dz of the bubble would require a force equal to

$$-\epsilon^2 \frac{\partial v_0}{\partial \epsilon} \frac{\partial}{\partial \epsilon}\left[\left(\rho_G + \frac{1}{2}\rho_L m_0\right) v_0\right] \frac{d\epsilon}{dz}$$

This force is usually negative, because the Richardson-Zaki exponent n is greater than unity and consequently the bubbles are attracted to regions of *larger* void fraction. This is a destabilizing force. If the swarm velocity is taken to be a constant, that is, independent of the gas holdup, we note that there is no destabilization.

The destabilizing force is opposed by the force arising from random motion of the bubbles

$$\left[\frac{\partial p_e}{\partial \epsilon} + \frac{12\pi a \mu_L}{\frac{4}{3}\pi a^3} \frac{1}{(1-\epsilon)^n} \delta_e\right] \frac{d\epsilon}{dz}$$

tending to equalize bubble voidage fluctuations. If the force which tends to drive the bubbles in the direction of larger void fraction is larger than the opposing force causing voidage equalization, the homogeneous bubbly regime is unstable and transition takes place to the churn-turbulent regime.

The Biesheuvel-Gorissen criterion (Eq. 6) for instability involves the following system parameters: ρ_G, ρ_L, μ_L, a, v_∞ and the exponent n. The bubble diameter, $d = 2a$, can be estimated from the correlation of Wilkinson et al. (1992):

$$\frac{d^2(\rho_L - \rho_G)g}{\sigma} = 8.8\left(\frac{U\mu_L}{\sigma}\right)^{-0.04}\left(\frac{\sigma^3 \rho_L}{g\mu_L^4}\right)^{-0.12}\left(\frac{\rho_L}{\rho_G}\right)^{0.22} \quad (9)$$

In an earlier study, Hoefsloot and Krishna (1993) have demonstrated that the Richardson-Zaki exponent n is the dominant parameter determining the stability of the system and, therefore, in the present study, we decided to experimentally establish the dependence of the Richardson-Zaki exponent n on gas density ρ_G for bubble columns.

Experimental Studies

Experiments were carried out in bubble columns of 0.05 and 0.1 m diameter made up of perspex sections; see Figure 4. The liquid phase used in the experiments was demineralized water.

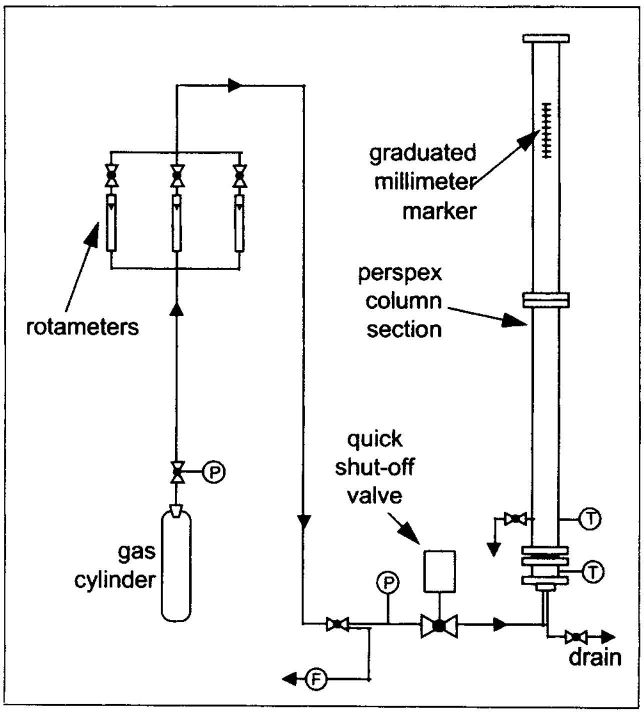

Figure 4. Setup used for the 0.1-m-diameter bubble column.

Table 1. Bubble-Column Configurations Used in Experimental Studies

Exp. Data Source	Column Dia. (m)	Liquid Phase	Ungassed Liquid Height (m)	Gas Phase	Pre. (bar)	Gas Distributor
This work	0.1 (0.05)	water	2.4 (1.2)	He	1	sintered plate
This work	0.05	water	1.2	50% He + 50% Air	1	sintered plate
This work	0.1 (0.5)	water	0.6, 1.2, 2.4 (1.2)	Air	1	sintered plate
This work	0.1 (0.05)	water	1.2 (1.2)	50% Air + 50% Ar	1	sintered plate
This work	0.1 (0.05)	water	1.2 (1.2)	Ar	1	sintered plate
This work	0.1 (0.05)	water	1.2 (1.2)	SF_6	1	sintered plate
Krishna (1992)	0.63	water	4	N_2	1	sparger
Koetsier et al. (1976)	0.05	water	0.6	He	1	sintered plate
Koetsier et al. (1976)	0.05	water	0.6	Ar	1	sintered plate
Wilkinson (1991)	0.16	water	1.5	N_2	10	sparger
Wilkinson (1991)	0.16	water	1.5	N_2	15	sparger
Wilkinson (1991)	0.16	water	1.5	N_2	20	sparger

A variety of gases were used in the experiments: He, 50% He-50% air, air, Ar, 50% air-50% Ar mixture and SF_6. The gas flow rate was carefully adjusted using one of a set of three rotameters in parallel. The rotameters were initially calibrated to obtain precise values for the gas flow rates. A glass sintered plate was used to distribute the gas uniformly into the column. The column temperatures were measured by means of two thermocouples placed near the bottom of the column. For a set gas flow rate, the column was allowed to reach steady state, and the dispersion height was measured using a graduated millimeter marking tape glued to the walls of the column and running the whole column length. The gas supply was shut off instantaneously using a quick shut-off valve, and the ungassed water height was recorded thereafter. The gas holdup could be determined by calculating the decrease in the column height on gas disengagement. The influence of the disengagement of gas in the plenum chamber was confirmed to be negligibly small. Experiments were carried out with ungassed liquid heights varying in the range 0.6–2.4 m. The superficial gas velocity used in the data analysis was based on pressure and temperature conditions prevailing at the bottom of the column. Table 1 summarizes the experimental conditions used. For each system duplicate experiments were carried out. Also given in Table 1 are published literature data sources for bubble column operation. All data collated from our own experiments and literature sources were used in the subsequent analysis.

On the basis of the experimentally determined U-ϵ data set, the bubble swarm velocity was determined from $V_{swarm} \equiv U/\epsilon$. The slope of an initial linear portion of the plot of $-\ln(V_{swarm})$ vs. $-\ln(1-\epsilon)$ yielded the value of $n-1$, while the intercept yielded the value of the single bubble rise velocity v_∞. A typical plot is shown in Figure 5 for SF_6 as the gas phase. At the regime transition point the linearity of the plot is destroyed sharply. The regime transition holdup ϵ_{trans} and velocity U_{trans} could thus be determined from the plot. Such plots were made for each data set listed in Table 1 and these were analyzed to obtain v_∞, n, ϵ_{trans} and U_{trans}.

Data Analysis and Results

Figure 6 shows that the single bubble rise velocity v_∞ does not depend on the gas density; in subsequent analysis a value $v_\infty = 0.25$ m/s is assumed. As seen in Figure 7, the Richardson-Zaki exponent n shows a pronounced decrease with increasing gas density; the data was correlated by:

$$n = 1 + 2.75\rho_G^{-0.54} \tag{10}$$

If n is large the bubbles "feel" each other much more than for a situation with small n. Explained another way, increasing the gas density reduces the interaction between bubbles. Increasing gas density should therefore make the regime of homogeneous bubbly flow more stable.

For the analogous situation of homogeneous gas-solids fluidization we have the relationship:

$$U = v_\infty \epsilon^n \tag{11}$$

where v_∞ is the single particle terminal velocity, and ϵ is the voidage. In a reference frame moving with the gas, the dis-

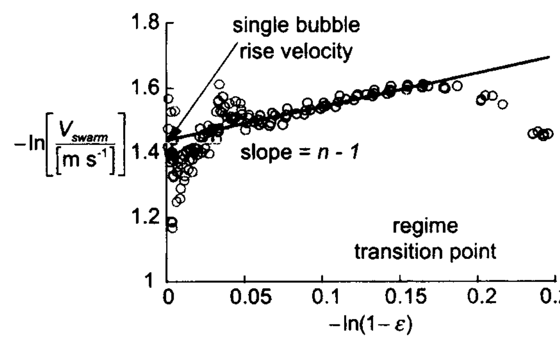

Figure 5. Typical experimental data obtained in 0.1-m-diameter column with SF_6 as gas phase and demineralized water as liquid phase.

Linear regression yields values for v_∞, the single bubble rise velocity, the Richardson-Zaki exponent and the regime transition holdup ϵ_{trans}.

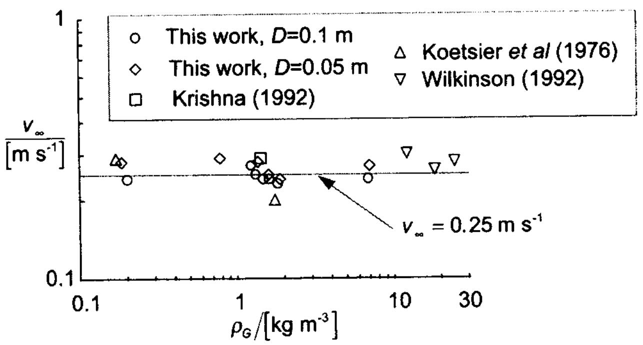

Figure 6. Single-bubble rise velocity as a function of gas density for the experimental investigations in Table 1.

persed (particle) phase swarm velocity V_{swarm} is related to the particle holdup (1-ϵ) by:

$$V_{\text{swarm}} = v_\infty (1-\epsilon)^{n-1} \tag{12}$$

The relation is analogous to Eq. 8. Analogous to our experimental findings above, Jacob and Weimer (1987) observed that for particulate expansion of fine carbon powders, the Richardson-Zaki index decreases with increasing pressure. Their experimental data further show that the minimum bubbling velocity $U_{mb}(\equiv U_{\text{trans}})$ increases with increasing system pressure in conformity with other experimental evidence for gas-solids fluidization (see Rowe et al., 1982).

The correlation (Eq. 10) for the Richardson-Zaki exponent n was then used in the Biesheuvel-Gorissen stability analysis to determine the values of ϵ_{trans} as a function of the gas density. The results are shown in Figure 8 and shows a significant increase in the transition gas holdup ϵ_{trans} with increasing gas density. The ϵ_{trans} values thus obtained represent an *upper bound* of the practically realizable transition holdups. In practice due, for example, to imperfect gas distribution, there will be an additional tendency towards destabilization of homogeneous bubbly flow, leading to an "earlier" transition to the heterogeneous flow regime. The experimentally determined values for ϵ_{trans} lie predominantly below those anticipated by the stability analysis, as indeed they should. Equation 3, developed by Wilkinson et al. (1992), predicts ϵ_{trans} values lying consistently below the modified stability analysis, as should be expected.

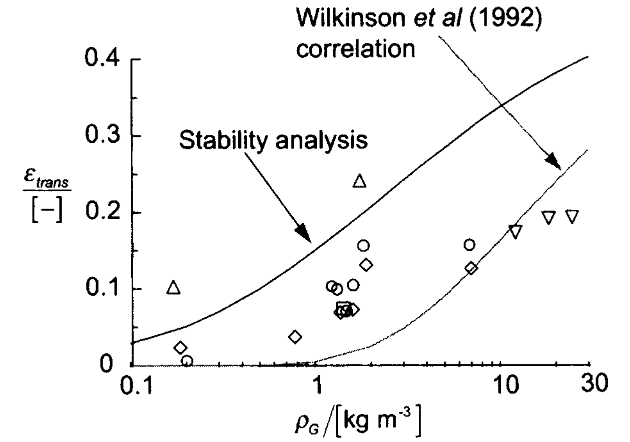

Figure 8. Transition gas holdup ϵ_{trans} as a function of gas density ρ_G.

Comparison of the values predicted by the stability analysis with the experimentally determined values and the Wilkinson et al. (1992) correlation, Eq. 3. The symbols are the same as in Figure 6.

Figure 9 shows the predictions of the velocity at the regime transition point $U_{\text{trans}} \equiv v_\infty \epsilon_{\text{trans}}(1-\epsilon_{\text{trans}})^{n-1}$ as a function of the gas density. Predictions of the modified stability analysis lie predominantly above the experimentally determined values and also above the values from the correlation of Wilkinson et al. (1992). The stability analysis provides a physical rationalization of the experimental observations.

Model for Large Bubble Gas Holdup in Heterogeneous Flow Regime

We assume that in the heterogeneous flow regime the large

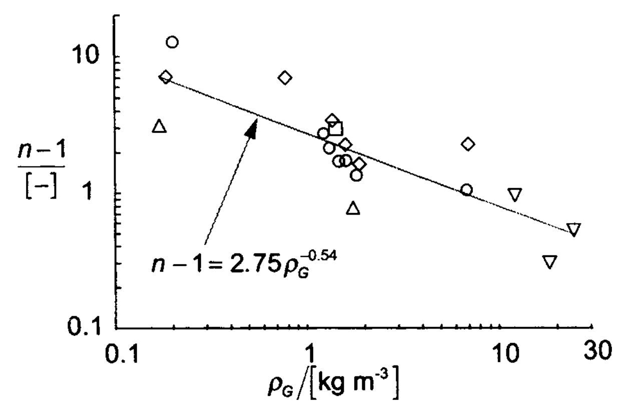

Figure 7. Richardson-Zaki exponent n as a function of gas density for the experimental investigations in Table 1.

The symbols are the same as in Figure 6.

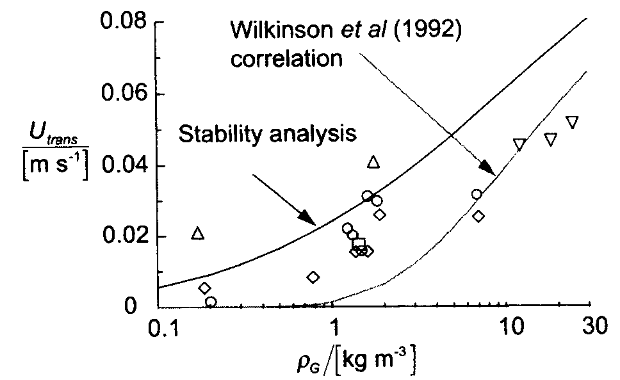

Figure 9. Transition gas velocity U_{trans} as a function of gas density ρ_G.

Comparison of the values predicted by the stability analysis with the experimentally determined values and those predicted by the Wilkinson et al. (1992) correlation, Eqs. 2-4. The symbols are the same as in Figure 6.

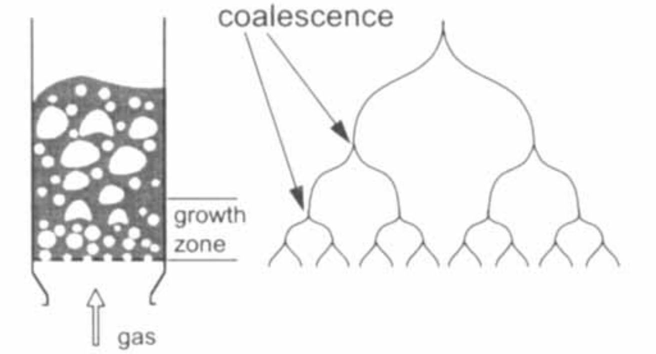

Figure 10. Darton's model for bubble growth in a gas-solid fluid bed.

bubbles are formed in the short region above the distributor by coalescence of small bubbles. The coalescence of small bubbles is pictured in Figure 10, which is based on the model of Darton et al. (1977), developed for bubbling gas-solid fluidized beds. In the Darton model, coalescence occurs between bubbles of neighboring streams, and the distance traveled by the bubbles between coalescence is proportional to their horizontal separation from neighboring bubbles. The application of the Darton model leads to the following relation for the bubble diameter:

$$d \propto (U - U_{\rm trans})^{2/5}(h + h_0)^{4/5} \tag{13}$$

where h is the height above the distributor. The parameter h_0 determines the initial bubble size formed at the distributor.

The coalescence process does not continue indefinitely, but at a certain height above the distributor, h^*, the equilibrium large bubble size is reached. From visual observations in our experimental studies we concluded that the equilibration height h^* is of the order of 0.1 m. At dispersion heights greater than h^*, the large bubbles coexist with the small bubbles in a dynamic equilibrium. The equilibrium large-bubble size is, therefore,

$$d_{\rm large} \propto (U - U_{\rm trans})^{2/5}(h^* + h_0)^{4/5} \tag{14}$$

The large bubbles in the bubble column are analogous to bubbles in gas-solid fluid beds (Krishna, 1993), and their rise velocity may be expected to follow a relationship of the form

$$U_{\rm large} \propto \sqrt{g d_{\rm large}} \tag{15}$$

The large bubble gas holdup should be expected to show the dependence:

$$\epsilon_{\rm large} \equiv \frac{(U - U_{\rm trans})}{U_{\rm large}} \propto (U - U_{\rm trans})^{4/5} \tag{16}$$

The four-fifths power dependence is in broad agreement with the Wilkinson et al. (1992) correlation, see Figure 5.

A more convincing evidence in support of Eq. 16 is provided by the data reported by Grund et al. (1988, 1992) who determined $\epsilon_{\rm large}$ using the dynamic gas disengagement technique.

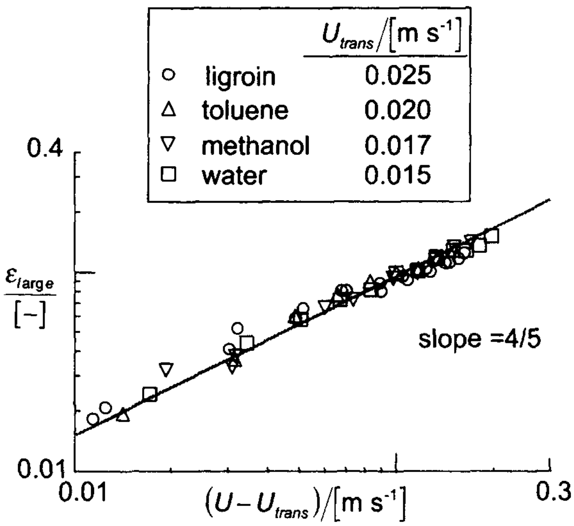

Figure 11. Log-log plot of the large-bubble holdup $\epsilon_{\rm large}$ vs. $(U - U_{\rm trans})$.

The transition gas velocity values used are: water, 0.015; methanol, 0.017; toluene, 0.020; ligroin, 0.025 m/s. Data source: Grund et al. (1988, 1992).

They performed experiments with water, toluene, methanol and ligroin as the liquid phase. We estimated the transition gas velocity $U_{\rm trans}$ from the experimental data and plotted $\epsilon_{\rm large}$ vs. $(U - U_{\rm trans})$, as shown in Figure 11. It is remarkable to note that all large bubble holdup data fall on the same line showing that the large bubble holdup is virtually independent of the liquid phase properties. The virtual independence of the large bubble holdup on the liquid phase properties has been noted earlier (Grund et al., 1992). Krishna et al. (1991) had demonstrated earlier that the large bubble holdup is also independent of the gas density.

Concluding Remarks

On the basis of the insight obtained in this article, we suggest that in the homogeneous flow regime the gas holdup must be correlated in the form:

$$\epsilon = \frac{U}{V_{\rm swarm}}; \quad V_{\rm swarm} = v_\infty (1-\epsilon)^{n-1}; \quad U \le U_{\rm trans} \tag{17}$$

where the single bubble rise velocity v_∞ can be estimated from the Wilkinson correlation (Eq. 4) for the small bubble rise velocity. While we may use Eq. 10 for the dependence of n on the gas density, the influence of the liquid phase properties (μ_L, σ, ρ_L) on the Richardson-Zaki exponent deserves further investigation.

The Richardson-Zaki exponent n has been shown to be an important parameter determining the stability of homogeneous bubbly flows in a bubble column. With increasing gas density, the value of this exponent decreases; physically this means that increasing gas density results in reduced interaction between neighboring bubbles and, consequently, reduced chance of propagation of instabilities. The Biesheuvel-Gorissen stability analysis, modified to incorporate the influence of the gas density on the exponent n, provides upper bounds for $\epsilon_{\rm trans}$ and

U_{trans}. The influence of liquid-phase properties (μ_L, σ, ρ_L) on the values of ϵ_{trans} and U_{trans} needs further attention (Deckwer and Schumpe, 1993).

For the heterogeneous regime of operation we suggest that the gas-holdup correlation of the form:

$$\epsilon = \epsilon_{trans} + \epsilon_{large}; \quad \epsilon_{large} = A(U - U_{trans})^{4/5}; \quad U \geq U_{trans} \tag{18}$$

where the constant A is expected to be independent of the gas density and the liquid phase properties. Experimental data for the large bubble rise velocity of Schumpe and Grund (1986) and Grund et al. (1992) seem to suggest that the parameter A could be a function of the column diameter. This aspect needs further detailed investigation in view of its consequences in scaleup.

Notation

- a = radius of gas bubble, m
- A = constant in Eq. 18, dimensionless
- d = bubble diameter, m
- d_{large} = diameter of large bubbles, m
- g = acceleration due to gravity, 9.81 $m \cdot s^{-2}$
- h = height above the gas distributor, m
- h^* = height above the gas distributor where the bubbles reach their equilibrium large bubble size, m
- h_0 = parameter determining the initial bubble size at the distributor, m
- m_0 = voidage correction for the added mass term $m_0 = (1+2\epsilon)/(1-\epsilon)$, dimensionless
- n = Richardson-Zaki exponent, dimensionless
- p_e = kinetic contribution of the effective pressure $= \epsilon\{\rho_G + [(1/2)\,\rho_L m_0]\}\overline{\Delta v^2}$, $kg \cdot m^{-1} \cdot s^{-2}$
- U = superficial gas velocity, $m \cdot s^{-1}$
- U_{mb} = minimum bubbling velocity for gas-solid fluid bed, $m \cdot s^{-1}$
- U_{trans} = superficial gas velocity at regime transition, $m \cdot s^{-1}$
- v_0 = rise velocity of bubble swarm in the zero volume flux reference frame, $m \cdot s^{-1}$
- v_∞ = single bubble rise velocity or particle terminal velocity, $m \cdot s^{-1}$
- $\overline{\Delta v^2}$ = mean-square velocity fluctuation $\overline{\Delta v^2} = (\epsilon/\epsilon_{cp})[1-(\epsilon/\epsilon_{cp})]v_0^2$, $m^2 \cdot s^{-2}$
- V_{large} = rise velocity of the large bubble population, $m \cdot s^{-1}$
- V_{small} = rise velocity of the small bubble population, $m \cdot s^{-1}$
- V_{swarm} = rise velocity of bubble or particle swarm in the laboratory fixed reference frame, $m \cdot s^{-1}$
- z = vertical distance, m

Greek letters

- α = constant of proportionality; $\alpha = 1$, dimensionless
- δ_e = effective diffusivity of bubble swarm voidage $\delta_e = \alpha a [\overline{\Delta v^2}]^{1/2}$, $m^2 \cdot s^{-1}$
- ϵ = fractional holdup of gas bubbles; also voidage for homogeneous gas-solids fluidization, dimensionless
- ϵ_{cp} = fractional holdup of gas bubbles at closest packing, $\epsilon_{cp} = 0.62$, dimensionless
- ϵ_{trans} = gas holdup at the regime transition point, dimensionless
- ϵ_{large} = holdup of fast-rising large bubbles, dimensionless
- μ_L = liquid viscosity, $Pa \cdot s$
- ρ_G = density of gaseous phase, $kg \cdot m^{-3}$
- ρ_L = liquid density, $kg \cdot m^{-3}$
- σ = surface tension of the liquid phase, $N \cdot m^{-1}$

Subscripts

- cp = closet packing
- e = effective
- G = gas phase
- l = large bubbles
- L = liquid phase
- mb = minimum bubbling
- p = particle phase
- small = small bubbles in gas-liquid systems
- swarm = bubble or particle swarm
- trans = transition point
- ∞ = single bubble or particle parameter

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