Gas Holdup in Bubble Column Reactors Operating in the Churn-Turbulent Flow Regime

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A comprehensive experimental study of gas holdup in bubble columns of varying diameters, fitted with different distributor types, using several liquids is presented. Air was used as the gas phase. Experiments to test the influence of gas density were also carried out with He, Ar, and SF_6. A generalization of the two-phase model for gas–solid fluidized beds was used to interpret the experimental data where the "dilute" phase is identified with the "large" bubble population and the "dense" phase with the liquid phase where the "small" bubble population is entrained. Gas holdups in dilute and dense phases were determined from dynamic gas disengagement experiments.

In the churn-turbulent regime of operation, voidage of the gas in the dense phase was independent of the superficial gas velocity. Reilly et al.'s correlations for the gas holdup and superficial gas velocity at the regime transition point estimate the gas voidage of the dense phase and the superficial gas velocity well through this phase. Corresponding correlations of Wilkinson et al. significantly underpredict dense-phase parameters. The experiment showed that the dilute phase or large bubble holdup in bubble columns, operating at superficial gas velocities > 0.1 m/s, is independent of liquid properties, how the gas is distributed and the density of the gas phase. But it is affected significantly by the column diameter. Relying on hydrodynamic analogies with a gas–solid–fluid bed, a simple correlation was developed that is considerably more accurate than the Wilkinson correlation that significantly overpredicts large bubble holdup.

Introduction

Bubble column reactors are often operated in the heterogeneous flow regime at high gas throughputs (typically higher than 0.1 m/s), high pressures (gas densities approaching 20 kg/m^3), and in columns of large diameters (approaching 6 m). In the heterogeneous, or churn-turbulent, flow regime the gas-phase conversion is largely dictated by fast-rising "large" bubbles, and for reactor design purposes it is important to be able to predict the large bubble holdup and the corresponding rise velocities.

Wilkinson et al. (1992) presented the following correlation for estimation of the large bubble holdup and total gas holdup:

$$\epsilon_b = \frac{(U - U_{\text{trans}})}{V_b}; \qquad \epsilon = \epsilon_{\text{small}} + \epsilon_b, \tag{1}$$

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where the gas velocity and holdup at the regime transition point are

$$U_{\text{trans}} = \epsilon_{\text{trans}} V_{\text{small}}; \qquad \epsilon_{\text{trans}} = 0.5\exp(-193\rho_G^{-0.61}\mu_L^{0.5}\sigma^{0.11}). \tag{2}$$

The rise velocities of the small and large bubble populations are correlated as

$$\frac{V_{\text{small}}\,\mu_L}{\sigma} = 2.25\left(\frac{\sigma^3\rho_L}{g\mu_L^4}\right)^{-0.273}\left(\frac{\rho_L}{\rho_G}\right)^{0.03} \tag{3}$$

and

$$\frac{V_b\,\mu_L}{\sigma} = \frac{V_{\text{small}}\,\mu_L}{\sigma} + 2.4\left(\frac{(U-U_{\text{trans}})\,\mu_L}{\sigma}\right)^{0.757}\left(\frac{\sigma^3\rho_L}{g\mu_L^4}\right)^{-0.077}\left(\frac{\rho_L}{\rho_G}\right)^{0.077}, \tag{4}$$

respectively. The Wilkinson correlation assumes that in the heterogeneous flow regime, $U > U_{\text{trans}}$, the small bubble holdup remains constant and its value equals the gas holdup at the regime transition point, $\epsilon_{\text{small}} = \epsilon_{\text{trans}}$.

The Wilkinson correlation anticipates the large bubble holdup to be influenced by gas density and liquid properties (density, viscosity, and surface tension) but uninfluenced by the column diameter D_T. It is to be noted that Wilkinson et al. (1992) did not actually measure the "large" bubble gas holdup, ϵ_b, and the correlations were obtained by fitting of experimental data on *total* gas holdup, ϵ, and assuming that the holdup of the "small" bubbles in the heterogeneous regime equals that at the regime transition point, ϵ_{trans}.

The objective of this work is to test the Wilkinson et al. correlations by direct comparison with experimental data on total, large and small bubble holdups and to put forward a reliable estimation procedure that can be used for scale-up.

Experimental Setup and Systems

Gas–liquid bubble column studies were carried out in columns of diameters 0.1, 0.174, 0.19, 0.38 and 0.63 m. A typical experimental setup is shown in Figure 1 for the 0.63-m-dia. bubble column. The column is made up of four polyacrylate sections with a total height of 4 m. The top of the column is connected to the exhaust. In all the experiments the pressure at the top of the column was nearly atmospheric. The 0.174-, 0.19- and 0.38-m-dia. columns were equipped with sintered bronze (with a mean pore size of 50 μm) plate gas distributors. The 0.63-m-dia. column was fitted with a spider-shaped sparger, Figure 1. The 0.1-m-dia. column was experimented with both a sintered glass plate distributor (with a mean pore size of 150–200 μm) and a polyacrylate sieve plate with 2.5-mm-dia. holes. Operating conditions used in various columns are in Table 1. The physical properties of various liquids used in the experiments are in Table 2. A total of 2,787 experiments were carried out.

For characterizing the hydrodynamics dynamic gas disengagement experiments were performed. This technique is well known and described in the literature (Daly et al., 1992). The dispersion height can be monitored by use of a pressure sensor connected to a PC for continuous recording of the voltage signals (these are proportional to the hydrostatic head). Figure 2 shows a typical disengagement experiment for the 0.63-m-dia. column with the air–water system. The initial fast disengagement is due to the escape of the "large" bubbles. When all the large bubbles have escaped, the disengagement profile is much less steep; now the "small" bubbles are disengaged. Appropriate corrections were applied to account for the disengagement of small bubbles during the initial step; such corrections have been well documented in the literature (see, e.g., Schumpe and Grund, 1986). The correction due to liquid downflow during the disengagement step was established as being negligible.

The total gas holdup ϵ and the large bubble holdup are determined from

$$\epsilon = \frac{H - H_0}{H}; \qquad \epsilon_b = \frac{H - H_1}{H}. \tag{5}$$

In this work we follow the approach of Ellenberger and Krishna (1994) for modeling of bubble column reactors and use a generalization of the two-phase theory for gas–solid fluid beds (Van Deemter, 1961; May, 1959) where the "dilute" phase is identified with the large bubble population and the "dense" phase with the liquid phase in which the small bubble population is entrained. The "dilute" phase holdup is thus ϵ_b. The voidage of gas in the dense phase ϵ_{df} is defined as

$$\epsilon_{df} \equiv \frac{\epsilon - \epsilon_b}{(1 - \epsilon_b)} = \frac{H_1 - H_0}{H_1}. \tag{6}$$

In the literature on bubble columns it is usual to define the

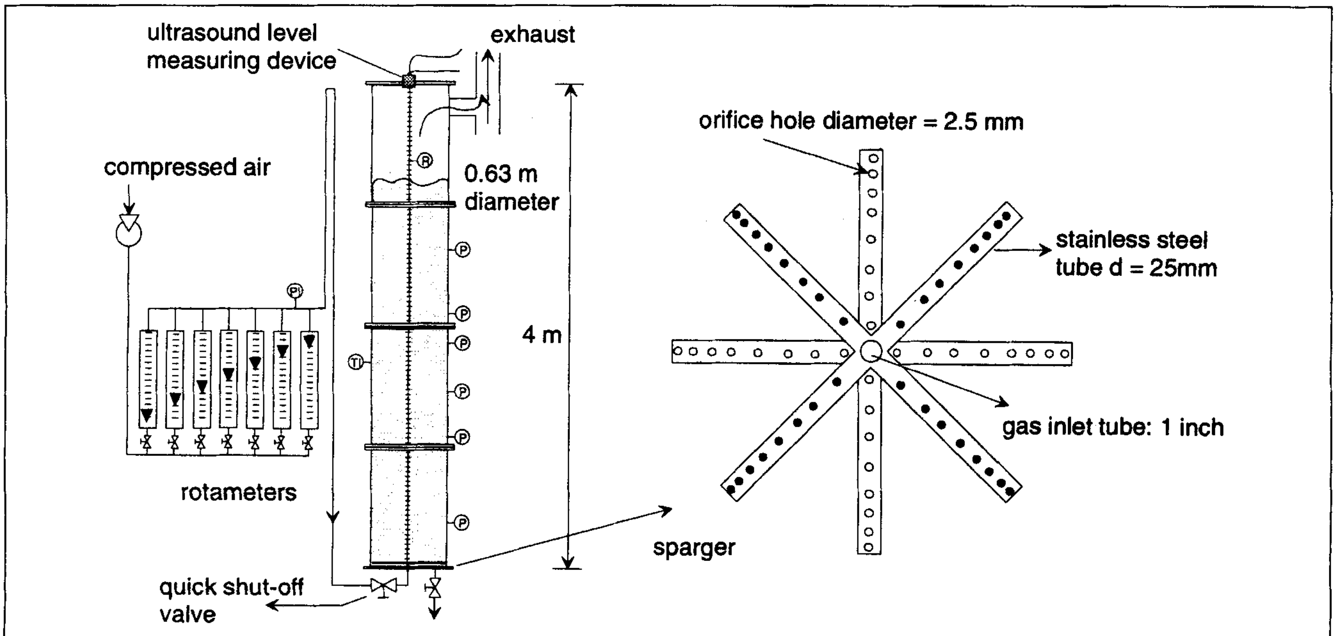

Figure 1. Experimental setup for 0.63-m-dia. gas–liquid bubble column fitted with spider sparger.

Table 1. Experimental Setup, Operating Conditions, and System Properties

D_T/m	Distributor	Systems Studied	Superficial Velocity m/s	Unexpanded Bed Height, H_0, m	No. of. exp.
0.63	Sparger (Figure 1)	air–water	0.010–0.366	0.5–2.2	346
		air–paraffin oil (A)	0.005–0.353	0.7	22
		air–paraffin oil (B)	0.009–0.307	1.9	60
		air–water + ⋯ + 100, 250, 500, and 1,000 ppm Separan	0.004–0.33	1.4–1.8	128
0.38	Bronze sintered plate	air–water	0.004–0.736	0.5–2	460
		air–paraffin oil (A)	0.002–0.682	0.9–1.6	239
0.19	Bronze sintered plate	air–water	0.001–0.664	0.4–1.2	209
		air–paraffin oil (A)	0.001–0.658	0.7–1.5	293
0.174	Bronze sintered plate	air–water	0.001–0.891	0.5–1.7	37
0.1	Glass sintered plate	air–water	0.001–0.391	1.1–1.3	153
		air–paraffin oil (A)	0.087–0.307	0.6–1.3	63
		air–tetradecane	0.001–0.17	0.3–1.2	185
		He–tetradecane	0.005–0.407	0.7–1.2	99
		Ar–tetradecane	0.001–0.249	0.7–1.2	119
		SF_6–tetradecane	0.005–0.075	0.7–1.2	70
0.1	Polyacrylate sieve plate	air–water	0.002–0.846	1.3–2.2	77
		air–paraffin oil (A)	0.001–0.844	1.3–1.6	64
		air–paraffin oil (B)	0.001–0.17	0.3–1.2	92
		air–tetradecane	0.001–0.866	1.3	71

small bubble holdup ϵ_{small} as (e.g., Deckwer and Schumpe, 1993)

$$\epsilon_{small} \equiv \epsilon - \epsilon_b = \frac{H_1 - H_0}{H}. \tag{7}$$

The slope of the second portion of the disengagement curve was used to determine the superficial gas velocity through the small bubble population, or dense phase, U_{df}, where we again adopt the terminology common to gas–solid fluid beds.

Total Gas Holdup, Small Bubble Holdup, and Dense-Phase Gas Voidage

Figure 3 shows a typical experimental result obtained for the total gas holdup ϵ, the small bubble holdup ϵ_{small}, and the "dense" phase gas voidage, ϵ_{df} for air/tetradecane in a 0.1-m-dia. column with sintered gas plate distributor. Beyond a superficial gas velocity of 0.1 m/s we note that the voidage of the dense phase ϵ_{df} is practically constant, whereas the small bubble holdup defined by Eq. 7 decreases somewhat with increasing U beyond 0.1 m/s. The constancy of the dense-phase gas voidage for $U > 0.1$ m/s was observed in all of our experiments; see, for example, the results for air/paraffin oil (A) and air/water in Figure 4. Also drawn in Figure 4 are the predictions of the gas voidage at the regime transition point according to the Wilkinson correlation, Eq. 2. We note that the measured values of the dense-phase gas voidage are significantly higher than the values of ϵ_{trans} from Eq. 2. The recent correlation of Reilly et al. (1994) for the transition point

$$\epsilon_{trans} = 0.59\, B^{1.5} \sqrt{\frac{\rho_G^{0.96}}{\rho_L}\sigma^{0.12}}; \qquad V_{small} = \frac{1}{2.84}\frac{1}{\rho_G^{0.04}}\sigma^{0.12};$$

$$U_{trans} = V_{small}\,\epsilon_{trans}(1 - \epsilon_{trans}), \tag{8}$$

Table 2. Physical Properties of Liquids Used

Liquid	Density kg/m³	Dynamic Viscosity mPa·s	Surface Tension mN/m (Est.)
Demineralized Water	998	1	72
Tetradecane	763	2.2	27
Paraffin oil (A)	795	2.3	28
Paraffin oil (B)	790	2.9	28
Polyacrylamide solutions (Separan AP-30)	998	Zero-shear viscosities: 50 (for 50 ppm solution) 100 (250 ppm) 190 (500 ppm) 350 (1,000 ppm)	72

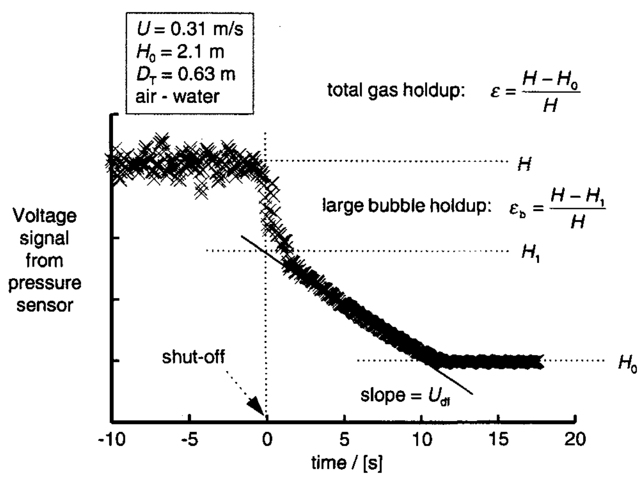

Figure 2. Typical dynamic gas disengagement experiment for 0.63-m-dia. column.

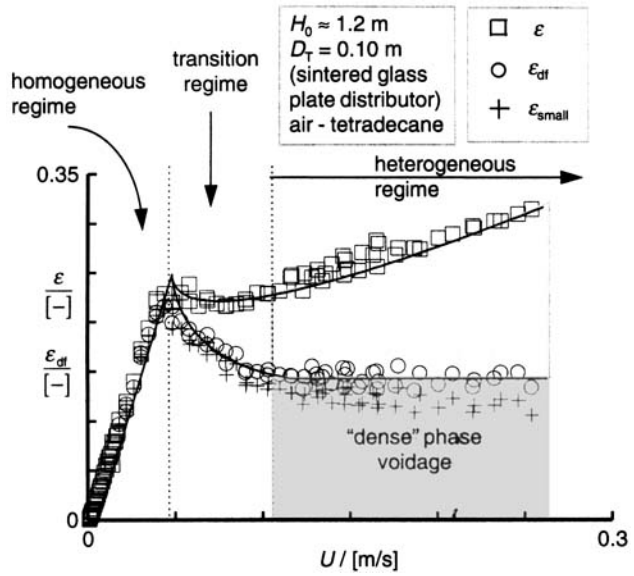

Figure 3. Total gas holdup, small bubble holdup, and dense-phase gas voidage as a function of the superficial gas velocity for air/tetradecane in a 0.1-m-dia. column.

where $B = 3.85$ provides a very good estimate of the dense-phase voidage for air/water and air/tetradecane; cf. Figure 4. The dense-phase gas voidage data presented in Figure 4c were obtained in a 0.1-m-dia. column fitted with glass sintered plate and with a sieve plate distributor. The values of the dense-phase gas voidage ϵ_{df} does depend on the method of gas distribution (see Figure 4); a more uniform gas distribution with a sintered plate distributor leads to a higher dense-phase gas voidage than with a sieve plate distributor.

With liquid mixtures such as paraffin oil (A), the transition gas holdup values ϵ_{trans} of both Reilly et al. (1994) and Wilkinson et al. (1992) underpredict the dense-phase voidage ϵ_{df}, with the Wilkinson correlation performing much worse than the Reilly correlation.

Increasing gas density increases the total gas holdup; see Figure 5. This increase is to be attributed to a delay in the regime transition point (Krishna et al., 1994). Measured values of the dense-phase gas voidage ϵ_{df} and the superficial gas velocity through the dense phase U_{df} are compared with the transition point predictions of Wilkinson et al. (1992) and Reilly et al. (1994) for air/tetradecane; see Figure 6. We note that the Wilkinson correlation severely underpredicts the values of the voidage and gas velocity through the dense phase, while the Reilly et al. predictions are reasonably good. For estimation purposes of ϵ_{df} and U_{df} we recommend the use of Eq. 8 assuming $\epsilon_{df} = \epsilon_{\text{trans}}$ and $U_{df} = U_{\text{trans}}$. One point to note, however, is that the Reilly correlation has been developed for a data set with a maximum value of $\epsilon_{\text{trans}} = 0.32$.

Large Bubble (Dilute-Phase) Holdup

Before presenting our experimental results we develop our model for estimation of the large bubble gas holdup. Large bubbles are pictured as being formed by coalescence of small bubbles. Drawing analogies with gas–solid fluidized beds, we adopt the model of Darton et al. (1977) to describe the coalescence process. In this model coalescence occurs between bubbles of neighboring streams, and the distance traveled by the bubbles before coalescence is proportional to their horizontal separation from neighboring bubbles. Following the Darton model, the diameter of a sphere having the same volume as the actual bubble is given by the relation:

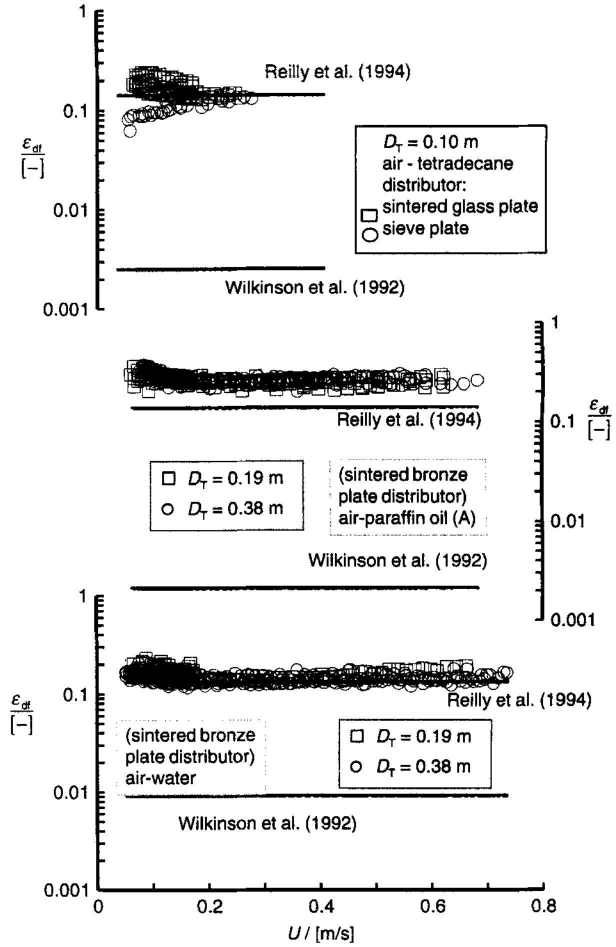

Figure 4. Dense-phase gas voidage as a function of the superficial gas velocity in the churn-turbulent regime of operation.

(a) Air/water; (b) air/paraffin oil (A); (c) air/tetradecane. The data in Figure 4c also demonstrate the influence of the gas distributor on the dense-phase gas voidage.

$$d_b = \alpha_1 (U - U_{df})^{2/5} (h + h_0)^{4/5} g^{-1/5}, \tag{9}$$

where α_1 is an empirical constant; and the parameter h_0 characterizes the distributor. For sintered plate distributors, the value of h_0 is 0.03 m (Darton et al., 1977). The initial bubble size formed at the distributor is thus:

$$d_{b0} = \alpha_1 (U - U_{df})^{2/5} (h_0)^{4/5} g^{-1/5}. \tag{10}$$

A short distance h^* above the distributor plate the bubbles reach their equilibrium size:

$$d_b^* = \alpha_1 (U - U_{df})^{2/5} (h^* + h_0)^{4/5} g^{-1/5} \quad \text{for} \quad h^* \le h \le H. \tag{11}$$

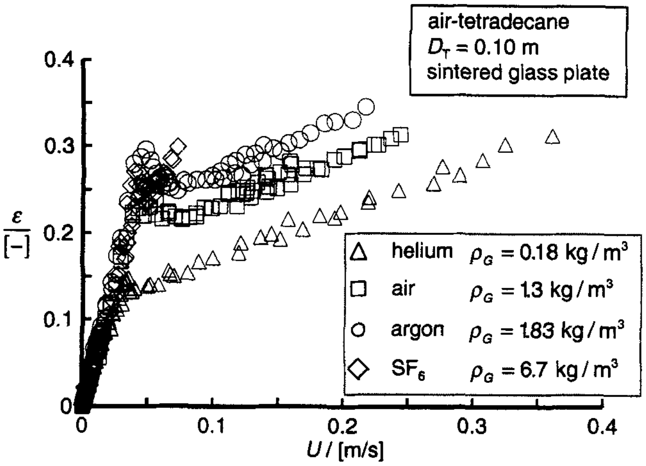

Figure 5. Influence of gas density on the total gas holdup for air/tetradecane.

The holdup of gas in the form of large bubbles in column of dispersion height H, can be calculated from the following relation

$$\epsilon_b = \frac{1}{H}\int_0^H \frac{(U - U_{df})}{V_b} dh, \tag{12}$$

where the rise velocity V_b is a function of the bubble diameter, d_b. The analysis of Jamialahmadi and Müller-Steinhagen (1993) shows that

$$V_b = \phi\sqrt{g d_b}, \tag{13}$$

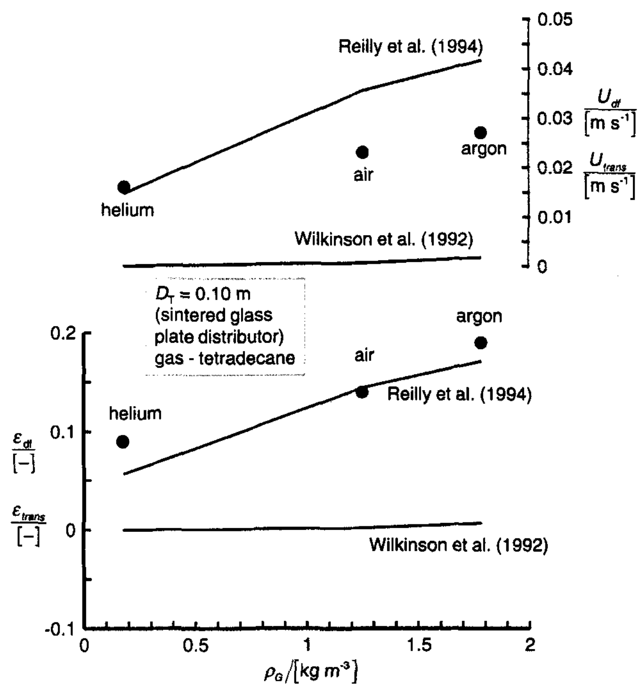

Figure 6. Influence of gas density on the (a) dense-phase gas voidage, and (b) superficial gas velocity through the dense phase with tetradecane as liquid phase.

where ϕ is an empirical constant. A similar relation is often used to describe the rise velocity of the dilute phase in gas–solid fluid (Werther, 1983). The constant ϕ is known to be a function of the column diameter. Following Werther (1983) we assert the empirical form of the dependence of ϕ on the column diameter

$$\phi = \phi_0 D_T^N. \tag{14}$$

Combining Eqs. 9–14, we can derive the following expression for the large bubble holdup in columns with dispersion heights $H > h^*$

$$\epsilon_b = \frac{1}{\sqrt{\alpha_1}\,\phi_0 D_T^n g^{2/5}} \frac{\left[(h^* + h_0)^{3/5} - (h_0)^{3/5}\right]}{(3/5)} \frac{(U - U_{df})^{4/5}}{H} + \frac{1}{\sqrt{\alpha_1}\,\phi_0 D_T^n g^{2/5}}(h^* + h_0)^{-2/5}(H - h^*)\frac{(U - U_{df})^{4/5}}{H}. \tag{15}$$

From visual observations it was established that the equilibration height h^* is of the order of 0.2–0.3 m, which is much smaller than the dispersion heights normally used in the bubble column reactors. Therefore, for total bed dispersion heights H greater say 1 m we obtain the further simplification of Eq. 15

$$\epsilon_b = \frac{1}{\sqrt{\alpha_1}\,\phi_0 D_T^N g^{2/5}}(h^* + h_0)^{-2/5}(U - U_{df})^{4/5} \quad \text{for} \quad H \gg h^*. \tag{16}$$

There is no *a priori* reason to assume that the equilibration height h^* would be independent of the superficial gas velocity through the large bubble phase $(U - U_{df})$. Assuming a power law dependence: $h^* = b0(U - U_{df})^{b1}$, we obtain after neglecting h_0 in comparison to h^*:

$$\epsilon_b = \alpha_2 \frac{1}{D_T^N} \frac{1}{(U - U_{df})^{b2}}(U - U_{df})^{4/5} \tag{17}$$

where

$$\alpha_2 = \frac{1}{\sqrt{\alpha_1}\,\phi_0 g^{2/5}}(b0)^{-2/5}; \qquad b_2 = \frac{2}{5}b1. \tag{18}$$

The model parameters in Eq. 17 are α_2, N, and $b2$.

From a detailed analysis, set by set, of the 2,787 experiments listed in Table 1 we established that the large bubble holdup for superficial gas velocities exceeding 0.1 m/s, was: (1) virtually independent of liquid properties, (2) virtually independent of the manner in which the gas is distributed, and (3) independent of the density of the gas phase. This implies that from the 2,787 experiments listed in Table 1, we can fit the model parameters α_2, N, and $b2$ after selecting only experimental large bubble gas holdup ϵ_b vs. $(U - U_{df})$ data values corresponding to dispersion heights H greater than 1 m and for superficial gas velocities U exceeding 0.1 m/s. The final form of our fitted correlation is

$$\epsilon_b = 0.268 \frac{1}{D_T^{0.18}} \frac{1}{(U - U_{df})^{0.22}} (U - U_{df})^{4/5}. \qquad (19)$$

It remains to verify this model by direct "confrontation" with our experimental data, taken set by set from Table 1. The transition velocity relation of Reilly et al. (1994), Eq. 8, provides a good estimate for U_{df}.

Influence of diameter of gas–liquid bubble column

The significant influence of the column diameter on the holdup of the large bubbles is illustrated in Figure 7 for air/paraffin oil (A) and air/water systems. Also shown in Figure 7 is the prediction of the large bubble holdup using the (1) Wilkinson correlation (Eqs. 1–4), taking $U_{df} = U_{trans}$, and (2) Eq. 19. The Wilkinson correlation significantly overpredicts the large bubble holdup and takes no account of the significant column diameter influence. The influence of column diameter on the large bubble holdup is consonant with the earlier findings of Grund et al. (1992).

Influence of liquid properties in gas–liquid bubble column operation

Variation of liquid properties has a very significant influence on the *total* gas holdup in bubble column operation, Figure 8a, 8b and 8c. The corresponding influence on the holdup of the *large bubbles* is, however, of insignificant importance; compare, one by one, with Figure 9a, 9b, and 9c. This independence of the large bubble holdup on liquid properties is a particularly useful and simple result, which has also been observed earlier by Grund et al. (1992).

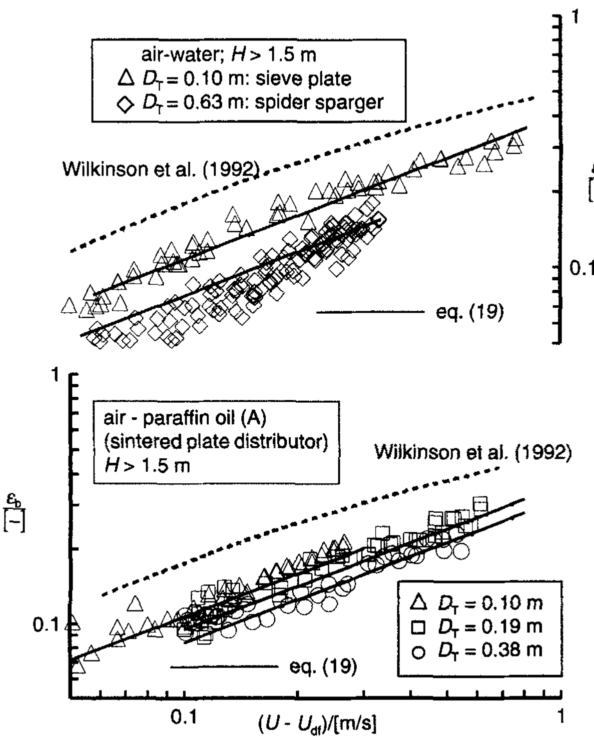

Figure 7. Influence of column diameter on large bubble holdup (a) air/paraffin oil (A) and (b) air/water systems. In the model calculations take $U_{df} = U_{trans}$; the experimental data points are plotted with measured U_{df} values.

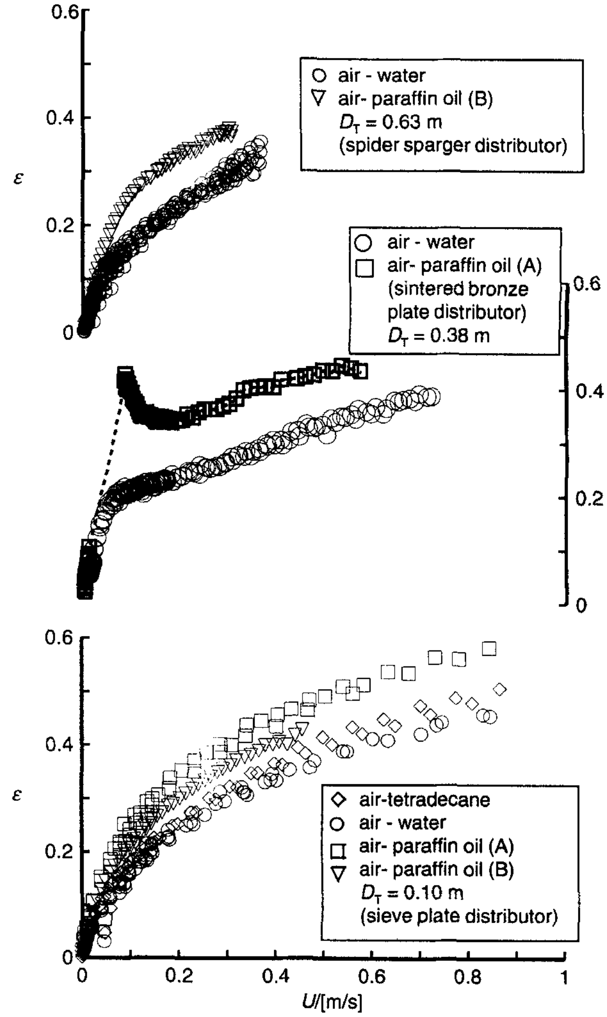

Figure 8. Influence of liquid properties on total gas holdup in (a) 0.1-m, (b) 0.38-m, and (c) 0.63-m-dia. columns.

Influence of gas distributor in gas–liquid bubble column operation

The manner in which gas is distributed has an important effect on the dense-phase gas voidage ϵ_{df} and on the *total* gas holdup ϵ; see data in Figures 4c and 10a for air/tetradecane obtained in a 0.1-m-dia. column fitted with either a glass sintered plate or a sieve plate distribution device. When we compare the *large bubble* gas holdups with different distributors, we note that the influence of the manner in which gas is distributed is insignificant; compare Figures 10a and 10b.

Influence of gas density in gas–liquid bubble column operation

While increasing gas density significantly increases the dense-phase gas voidage (cf. Figure 6), it has hardly any influence on the holdup of the large bubbles; see Figure 11. This independence of the large bubble gas holdup on the gas distributor is a useful and simple result that holds for gas

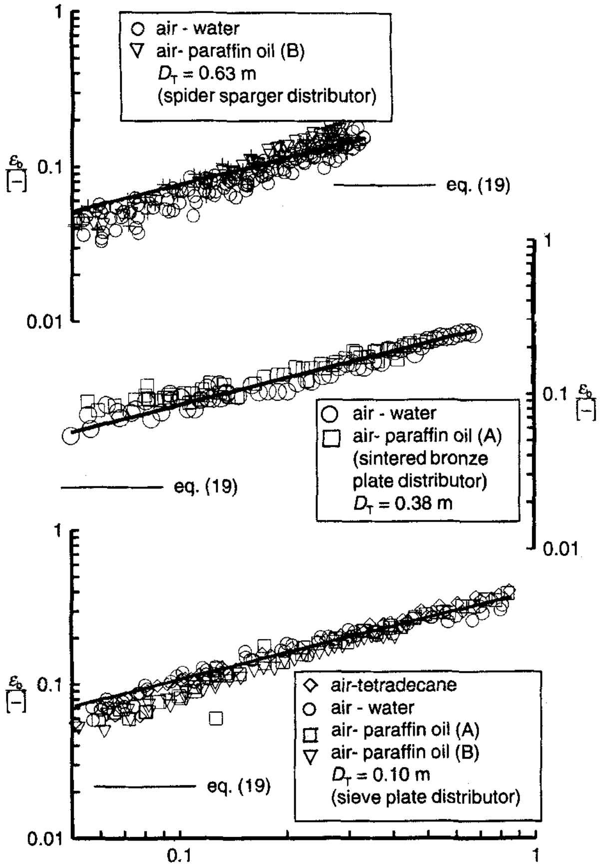

Figure 9. Large bubble holdup in (a) 0.1-, (b) 0.38-, and (c) 0.63-m-dia. columns for various liquids.

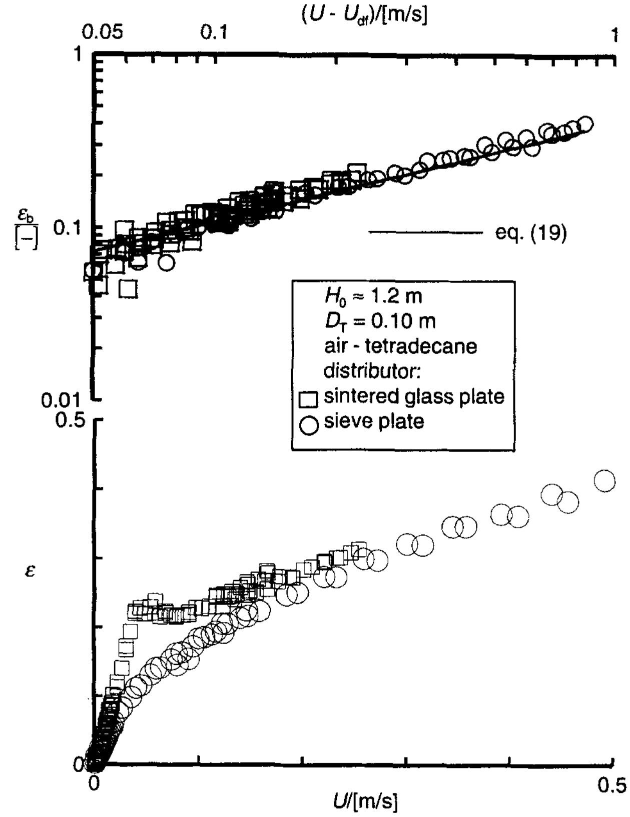

Figure 10. Influence of gas distributor on (a) total gas holdup, and (b) large bubble holdup in 0.1-m-dia. bubble column with tetradecane as liquid phase.

densities ranging to 6.7 kg/m^3. The validity of this conclusion needs to be tested for higher gas densities. Krishna et al. (1991) interpreted their high pressure bubble column data ranging to 20 bar, assuming that the large bubble gas holdup is independent of the system pressure. In Figure 11 we have also plotted the large bubble holdup predictions of the Wilkinson correlation for He and SF_6 as the gaseous phase; we note that the Wilkinson correlation predicts an exaggerated influence of gas density on ϵ_b.

Statistical Comparison of Wilkinson Correlation with Our Model

For our complete data set from Table 1 for operation in the churn-turbulent flow regime, consisting of 1,735 data points we compared the predictions of the large bubble holdup using the Wilkinson correlation, Eqs. 1–4 with our model, Eq. 19, taking the Reilly correlation for the transition gas velocity as estimate for U_{df}. The values of the average relative deviation, δ, are compared in Table 3, and this shows the superiority of our model. The Wilkinson consistently overpredicts the large bubble holdup while significantly underpredicting the small bubble holdup.

For the prediction of the total gas holdup following Wilkinson we have $\epsilon = \epsilon_b + \epsilon_{small}$, with $\epsilon_{small} = \epsilon_{trans}$, estimated from Eq. 2. Our approach to predicting the total gas holdup in the churn-turbulent flow assuming the constancy of the dense-phase voidage ϵ_{df} leads to

$$\epsilon = \epsilon_b + \epsilon_{df}(1 - \epsilon_b), \tag{20}$$

where we take $\epsilon_{df} = \epsilon_{trans}$, estimated from Eq. 8. The average relative deviations of the total gas holdup predictions are also given in Table 3. We note that the Wilkinson correlation

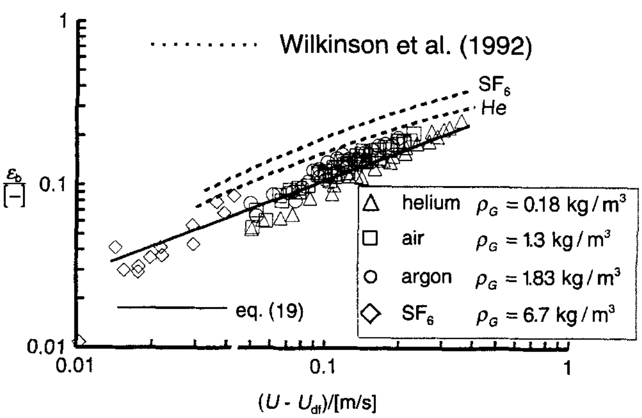

Figure 11. Influence of gas density on large bubble holdup in 0.1-m-dia. bubble column with tetradecane as liquid phase.

In the model calculations take $U_{df} = U_{trans}$; the experimental data points are plotted with measured U_{df} values.

Table 3. Statistical Comparison of Our Model with Wilkinson Correlation

Data Set	No. of Data Points ND	δ for ϵ_b Predictions: Wilkinson Eqs. 1–4	δ for ϵ_b Predictions: Our Model Eqs. 8 and 19	δ for ϵ Predictions: Wilkinson Eqs. 1–4	δ for ϵ Predictions: Our Model Eqs. 8, 19, and 20
This work (Table 1)	1735	1.25	0.16	0.24	0.23
Wezorke (1986)	241	1.48	0.24		

$$\delta = \frac{1}{ND}\sum_{i}^{ND}\left|\frac{\exp_i - \mathrm{pred}_i}{\exp_i}\right|$$

does well as our model; this is to be understood because the Wilkinson correlation was set up by fitting almost as experimental data on the total gas holdup.

We also culled large bubble holdup data from the thesis of Wezorke (1986), who made extensive measurements in columns of 0.09, 0.19, 0.305 and 0.441 m diameter and with water, ethylene glycol, and propylene glycol as the liquid phase. The average relative deviation values for the Wilkinson correlation are again significantly higher than the predictions of our model.

Conclusions

For operation at superficial gas velocities exceeding 0.1 m/s, the large bubble holdup in bubble column reactors is affected significantly by the column diameter and is virtually unaffected by the physical properties of the gas and liquid phases. Further, the manner in which the gas is distributed is unimportant. The correlation developed in this article, Eq. 19, is much simpler and provides much more accurate estimations of the large bubble holdup than the Wilkinson et al. (1992) correlation. The Wilkinson correlation generally tends to overpredict the values of the large bubble holdup while underpredicting the small bubble holdup. We recommend Eq. 19 for the estimation of the large bubble holdup. The Reilly correlation, Eq. 8, is recommended for the estimation of the transition parameters ϵ_{trans} and U_{trans}; these parameters provide good estimates of the ϵ_{df} and U_{df}. The total gas holdup can be estimated from Eq. 20.

Notation

$b0$ = fit parameter for equilibration height
$b1$ = fit parameter for equilibration height
B = constant in the Reilly correlation, Eq. 8
d_{b0} = initial bubble size formed at distributor, m
d_b^* = equilibrium bubble size, m
g = acceleration due to gravity, 9.81 $m \cdot s^{-2}$
h = height above the gas distributor, m
H_0 = height of ungassed bed, m
H_1 = height of dispersion after escape of large bubbles, m
N = power in the rise velocity correlation
ND = number of data points
U = superficial gas velocity, $m \cdot s^{-1}$
$(U - U_{df})$ = superficial gas velocity through the large bubbles, $m \cdot s^{-1}$
U_{df} = superficial velocity of gas through the small bubbles, $m \cdot s^{-1}$
δ = average relative deviation
μ_G = gas viscosity, Pa·s
μ_L = liquid viscosity, Pa·s
ρ_G = density of gaseous phase, $kg \cdot m^{-3}$
ρ_L = liquid density, $kg \cdot m^{-3}$
σ = surface tension of liquid phase, $N \cdot m^{-1}$

Subscripts

G = gas phase
L = liquid phase
0 = conditions at the gas distributor ($h = 0$)

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