

Liquid phase dispersion in bubble columns operating in the churn-turbulent flow regime

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Abstract

The objective of this paper is to develop a reliable, yet-simple, correlation for the liquid phase axial dispersion coefficient $D_{ax,L}$ in bubble column reactors. In order to develop this correlation we carried out measurements of the centre-line liquid velocity $V_L(0)$ and $D_{ax,L}$ in bubble columns of diameter $D_T=0.174, 0.38$ and 0.63 m with the air-water system. The measurements were performed in the churn-turbulent regime of operation with superficial gas velocities U in the range $0.05-0.35$ m/s. From the several literature correlations tested, the one proposed by Riquarts [Chem. Ing. Techn. 53, 1981, 60-61] can be recommended for estimation of $V_L(0)$ for air-water systems. The Riquarts correlation however anticipates the dependence of $V_L(0)$ on the kinematic viscosity of the liquid phase. Measurements of $V_L(0)$ were also carried out in the 0.38 m diameter column with Tellus oil as the liquid phase, with a kinematic viscosity 87 times that of water. Comparison of $V_L(0)$ measurements with water and Tellus oil shows a negligible influence of liquid viscosity, leading us to conclude that the best estimation procedure for $V_L(0)$ is to use the Riquarts correlation with properties of the water as the liquid phase. The above conclusions are also supported by Eulerian simulations of the bubble column hydrodynamics.

Our measured data on the $D_{ax,L}$ and $V_L(0)$ were combined to develop a simple correlation: $D_{ax,L}=0.31 V_L(0) D_T$; this correlation also provides a good description of experimental data from the literature. © 2000 Elsevier Science S.A.

Keywords: Bubble columns; Churn-turbulent flow regime; Bubble rise velocity; Radial velocity profiles; Column diameter influence; Axial dispersion coefficient

1. Introduction

Bubble column reactors operated in industry have several distinguishing features: (1) large column diameters are involved, ranging to 6 m, (2) high superficial gas velocities, in the $0.1-0.4$ m/s range, are usually used, (3) the system pressure can range to 6 MPa and (4) the liquid phase often consists of a non-aqueous hydrocarbon mixture [1–2]. Laboratory studies on bubble column hydrodynamics are usually carried out with the air-water system, at ambient pressure conditions, in columns that are smaller than 0.5 m in diameter and at superficial gas velocities below 0.1 m/s [3–7].

The estimation of the axial dispersion coefficient of the liquid phase, $D_{ax,L}$, is important for the design and scale up of bubble column reactors and is the principal focus of this work. Literature correlations vary considerably in complexity and detail [3–15]. All correlations anticipate a significant increase in $D_{ax,L}$ with increasing column diameter D_T , often correlated as a power-law dependence D_T^n . The value of the power law index n varies between 1 and 1.5. The

dependence of $D_{ax,L}$ on the superficial gas velocity U usually ranges within $U^{0.25}-U^{0.5}$. Several correlations for liquid phase axial dispersion coefficient [11,13,15] express $D_{ax,L}$ as a product of the liquid circulation velocity and the column diameter D_T . The upwardly directed axial-component of the liquid velocity at the centre of the column, $V_L(0)$, can be taken as a measure of the liquid circulation velocity for bubble columns in which there is no net inflow or outflow of liquid. There are several correlations in the literature for $V_L(0)$ and these show considerable differences in their dependence on the column diameter. The wide variations in the predictions of $V_L(0)$ as a function of U and D_T are underlined in the calculations presented in Fig. 1 for the air-water system. We note from Fig. 1a that for a 0.63 m column operating at $U=0.35$ m/s the predictions of the centre-line liquid velocity $V_L(0)$ found for the various correlations [11,15–22] vary from 0.85 to 1.4 m/s. From Fig. 1b we note that for a 6 m diameter column operating at $U=0.30$ m/s the predictions of $V_L(0)$ varies from 0.9 to 4.5 m/s. This represents a variation of a factor of five. The reason for this wide variation in the predictions for $V_L(0)$ is that the experimental studies on which these correlations have been based are limited in the range of column diameters studied. A further point to note

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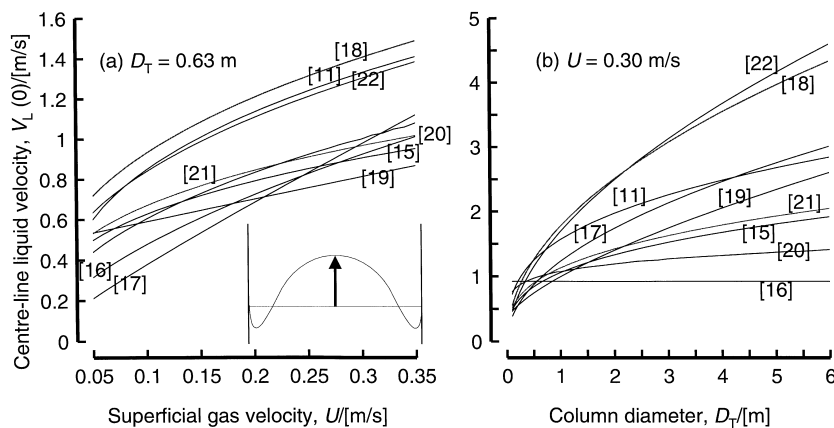


Fig. 1. Comparison of literature correlations for the centre-line velocity $V_L(0)$ for air-water system. (a) Variation $V_L(0)$ with superficial gas velocity for a column of 0.63 m diameter. (b) Variation of $V_L(0)$ with column diameter for a superficial gas velocity of 0.3 m/s. The numbers in the brackets refer to the reference number of the papers.

is that some correlations [18,20] anticipate a dependence of $V_L(0)$ on the liquid phase kinematic viscosity ν_L whereas most other correlations do not include such a dependence.

In order to develop a reliable correlation for the axial dispersion coefficient $D_{ax,L}$ we have undertaken an experimental study in which we measure both $V_L(0)$ and $D_{ax,L}$ in columns of three different diameters operating in the churn-turbulent flow regime. In order to investigate the influence of the liquid viscosity we have also carried out measurements with Tellus oil as the liquid phase, which has a kinematic viscosity 87 times that of water. In order to verify the scale up procedure to commercial scale columns we have

also performed Eulerian simulations of the bubble column hydrodynamics with air-water and air-Tellus oil systems in a variety of columns with diameters ranging from 0.174 to 6 m.

2. Experimental

Three different column diameters were used in the experiments, $D_T = 0.174, 0.38$ and 0.63 m. In each of these columns three types of experiments were performed: (1) measurement of the radial distribution of the axial component of the liquid velocity, $V_L(r)$, (2) measurement of the centre-line

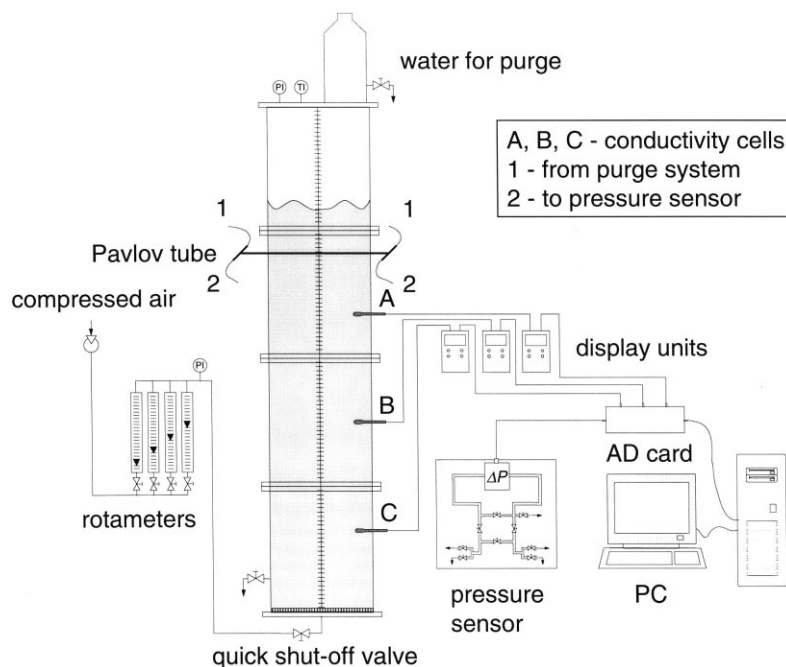


Fig. 2. Typical experimental set up for the 0.38 m diameter column showing the Pavlov tube and the three electrical conductivity sensors located along the height of the column.

liquid velocity, $V_L(0)$ and (3) measurement of the axial dispersion coefficient $D_{ax,L}$. All our measurements were carried out by operating the bubble columns in ‘batch mode’ with no inflow or outflow of liquid.

All three columns were made up of four polyacrylate sections with the total height of 4 m. In all three columns the pressure at the top corresponded to ambient conditions (101.3 kPa). A typical experimental set-up is shown in Fig. 2 for the 0.38 m bubble column. The 0.174 and 0.38 m diameter columns were equipped with sintered bronze plate gas distributors with an average pore size of 50 μm . The 0.63 m column was provided with a spider shaped sparger, described in earlier work [23]. The gas phase (air) was introduced at the bottom of the columns using different gas distributors. The liquid phase used in the experiments was either demineralized water ($\mu_L=0.001\text{ Pa s}$; $\rho_L=1000\text{ kg/m}^3$; $\sigma=0.072\text{ N/m}$) or Tellus oil ($\mu_L=0.075\text{ Pa s}$; $\rho_L=862\text{ kg/m}^3$; $\sigma=0.028\text{ N/m}$). The water properties were obtained from the Handbook of Chemistry and Physics. The density and viscosity of Tellus oil were obtained from the supplier’s catalogue. The surface tension of Tellus oil was estimated to equal that of petroleum fractions such as paraffin oil.

The axial component (upwardly-directed) of the liquid velocities along the radial positions at different superficial gas velocities were measured using a modified Pitot tube, also called ‘Pavlov tube’ [24]. The Pavlov tube consists of a cylindrical tube placed across the column, through two diametrically opposed holes; see Fig. 3a. The two 1 mm holes were drilled as close as possible to each other. The radial liquid profiles were measured by moving the tube along the column radius. The two holes were placed in two different planes, perpendicular (orthogonal) to each other. A diaphragm separated the interior of the tube in two sections, midway between the two holes; see Fig. 3a. During oper-

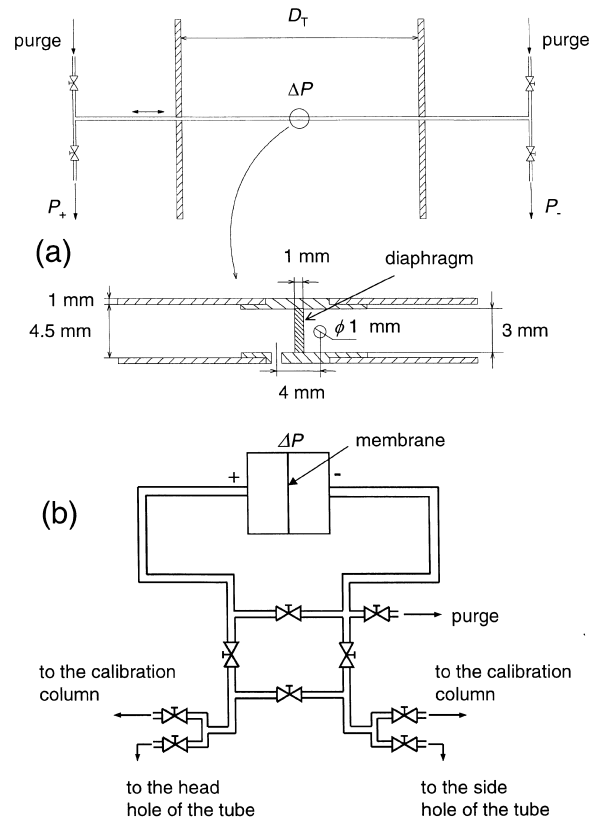


Fig. 3. Schematic diagram of (a) Pavlov tube and (b) differential pressure measuring system. The details of the Pavlov tube used for measuring the liquid velocity along the radius of the column are available on our web site: <http://ct-cr.4.chem.uva.nl/bc>.

ation of the column, bubbles can penetrate in the Pavlov tube, influencing the measurements. To minimise the disturbance of the measurements, a purge system was developed to remove the bubbles before each set of data acquisition.

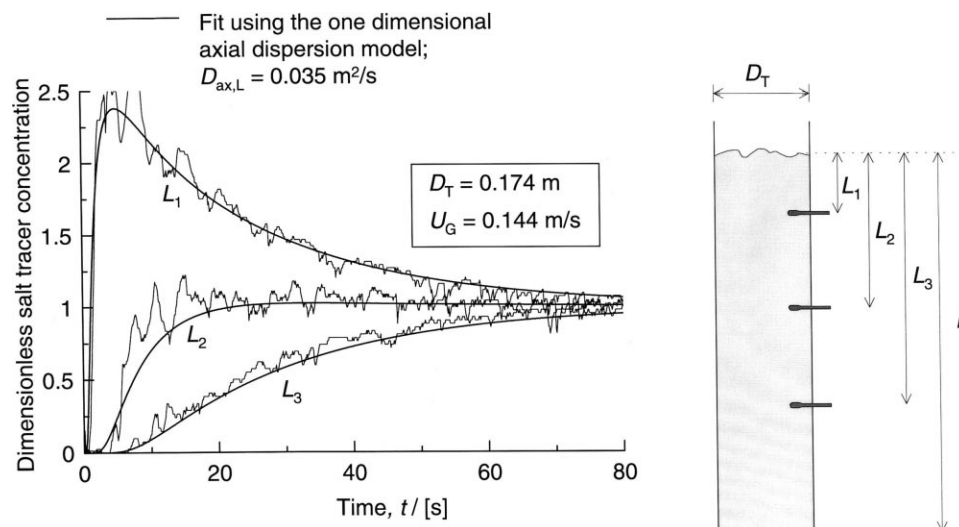


Fig. 4. Normalized liquid phase tracer concentrations measured at three different locations along the height of the column in response to pulse tracer injection. The smooth curves represent the fits to the curves from fitting a diffusion model described in Deckwer [3]. Details of the measurement technique, along with data analysis are available on our web site: <http://ct-cr4.chem.uva.nl/bc>.

Table 1

Column configurations, systems, operating conditions and grid details of CFD simulations. For operation at $U < U_{trans}$, homogeneous bubbly flow regime was taken to prevail. For operation at $U > U_{trans}$, the complete three phase model was invoked. The large bubble phase was injected over the central 13 (or 32) of the 30 (or 75) grid cells. The small bubble phase was injected over the central 24 (or 61) of the 30 (or 75) grid cells. The reported liquid velocity profiles are at the observation heights reported below

Liquid phase	Column diameter D_T (m)	Column height(m)	Initial liquid height (m)	Observation height (m)	No. of grid cells (radial)×(axial)	Superficial gas velocity, U /[m/s]	U_{trans} (m/s)
Water	0.174	3	1.8,2	1.6	30×160	0.02,0.034,0.09,0.16,0.23,0.27,0.3	0.034
Water	0.38	3	1.8, 2	1.6	30×160	0.02,0.034,0.09,0.16,0.23,0.285,0.3,0.35	0.034
Water	0.63	3	1.8, 2	1.6	30×160	0.02,0.034,0.09,0.16,0.23,0.27,0.3,0.35	0.034
Water	1	7	4	4	75×150	0.3	0.034
Water	1.5	8	4.8	4.8	75×410	0.3	0.034
Water	2	13,12	10,7	9.7	75×270	0.16, 0.3	0.034
Water	3	15	10	10	75×310	0.3	0.034
Water	4	25	20	18	75×510	0.16, 0.3	0.034
Water	6	35	25,20	18	75×710	0.16, 0.3	0.034
Tellus oil	0.1	2	1.4	1.4	30×110	0.05,0.09,0.125,0.16	0
Tellus oil	0.19	2	1.4	1.4	30×110	0.05,0.09,0.16,0.23,0.3,0.35	0
Tellus oil	0.38	2	1.4	1.4	30×110	0.05,0.09,0.125,0.16,0.23,0.3,0.35	0
Tellus oil	1.5	8	5.3	4.5	75×410	0.05,0.09,0.16	0
Tellus oil	2	13	10	9	75×270	0.16,0.3	0
Tellus oil	4	25	20	20	75×510	0.16,0.3	0
Tellus oil	6	35	20	20	75×710	0.16,0.3	0

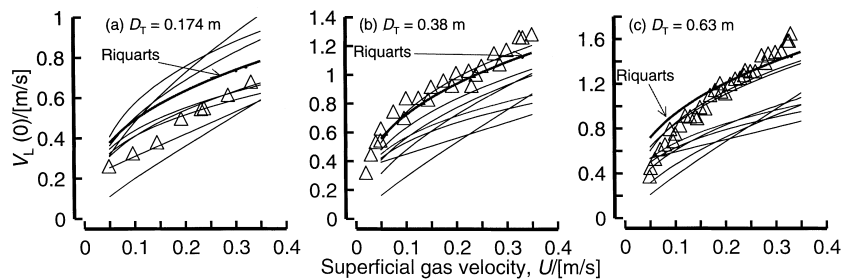


Fig. 5. Comparison of literature correlations for the centre-line velocity $V_L(0)$ for air–water system with experimental data.

The short measuring time of 1 min that was chosen reduces the penetration chances of the bubbles into the tube. After purging the Pavlov tube, the pressure differential measuring system (Fig. 3b) must stabilise and therefore a waiting time of 30 s was allowed for before the next data acquisition was made. The extremities of the tube were connected with

PVC tubes to the measuring system (Fig. 3b). The complete measuring system consists of a pressure sensor (Validyne DP15 transducer), a display and a personal computer (see Fig. 2).

For determination of the liquid phase residence time distribution a saturated solution of NaCl was used as a tracer.

Table 2

Standard deviation of literature correlations for centre-line velocity from experimental data

	$D_T=0.174$ m	$D_T=0.38$ m	$D_T=0.63$ m	Sum for all measured data
Air–water				
No. of data points measured in this work	8	24	35	67
Ohki and Inoue [16]	0.0505	0.0687	0.1903	0.3617
Ueyama and Miyauchi [17]	0.0176	0.1853	0.2079	0.4208
Joshi [11]	0.0579	0.0102	0.0143	0.1343
Riquarts [18]	0.0212	0.0091	0.0186	0.1245
Zehner [15]	0.0029	0.0958	0.1564	0.3411
Nottenkämper et al. [19]	0.0060	0.1532	0.2198	0.4128
Ulbrecht et al. [20]	0.0322	0.0458	0.1180	0.2862
Kawase and Moo-Young [21]	0.0045	0.0718	0.1213	0.2994
Bernemann [22]	0.0024	0.0250	0.0186	0.1376

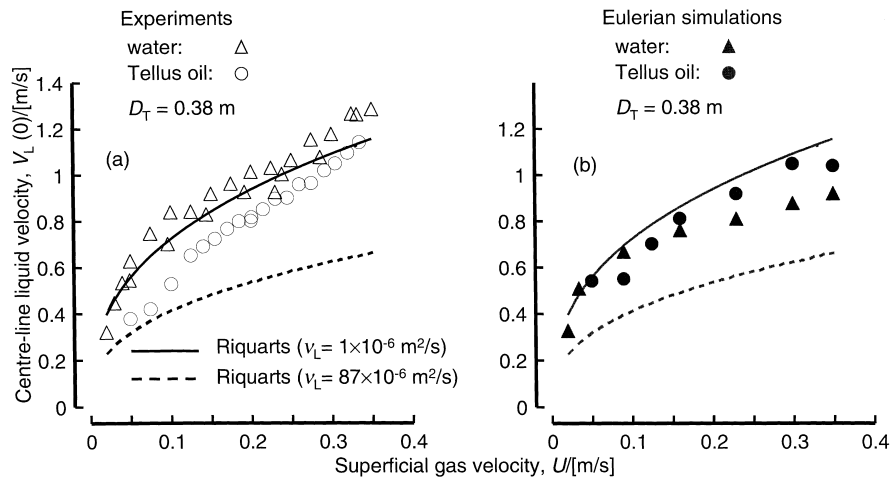


Fig. 6. (a) Comparison of experimental centre-line velocity data $V_L(0)$ for air–water and air–Tellus oil systems in 0.38 m diameter column with Riquarts correlation (1981). (b) Comparison of Eulerian simulations of $V_L(0)$ for air–water and air–Tellus oil systems in 0.38 m diameter column with Riquarts correlation [18]. Animations of the column start-up dynamics for the simulations can be viewed on our web site: <http://ct-cr4.chem.uva.nl/oil-water>.

The solution was injected into the batch liquid phase as a pulse just above the dispersion height. Different volumes of tracer were used, depending on gas velocity, column diameter and injection position in order to obtain the optimal signal. In the 0.63 m diameter column the tracer was injected both in the middle and near the wall. For the remaining two columns a single injection position was used, near the wall. The transient tracer concentration were monitored continuously by means of three Metrohm immersing-type conductivity cells which were placed near the wall, at different heights (A, B and C) as indicated in Fig. 2. Fig. 4 shows typical transient tracer concentrations for the 0.174 m diameter column; these signals were fitted using the analytic solution to the diffusion equation [3] to obtain a single value of the liquid phase axial dispersion coefficient, $D_{ax,L}$ for a given experiment. The position at which the tracer was injected had no influence on the fitted values of $D_{ax,L}$.

A detailed description of the experimental set-ups, data analysis of the signals from the Pavlov tube and electrical conductivity cells, along with the underlying theory are available in Urseanu [25] and also on our website: <http://ct-cr4.chem.uva.nl/bc>.

2.1. Eulerian simulations of bubble column hydrodynamics

In addition to the experimental work Eulerian simulations of bubble columns operating in the churn-turbulent flow regime were carried out by identifying three phases: (1) liquid, (2) ‘small’ bubbles and (3) ‘large’ bubbles. The momentum exchange, or drag, coefficients, for the small bubbles are determined from standard literature correlation [26]. The drag coefficient for the large bubbles are determined using the procedure developed by Krishna et al. [27]. Full details of the Eulerian simulation model are available elsewhere [28]. Several simulations assuming cylindrical axisymmetry

were carried out for a range of column diameters for both air–water and air–Tellus oil; details are given in Table 1.

2.2. Centre-line liquid velocity measurements $V_L(0)$

The measured centre-line velocities $V_L(0)$ with the air–water system are compared in Fig. 5 with literature correlations [11,15–22]. For the complete experimental data

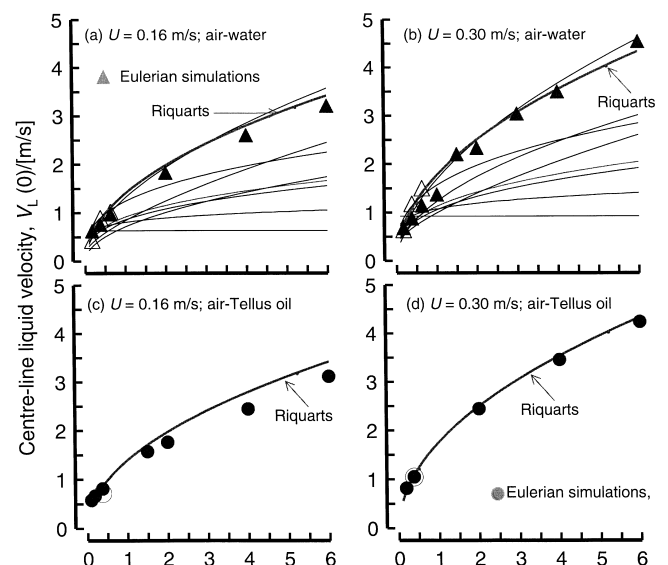


Fig. 7. Variation $V_L(0)$ with column diameter for superficial gas velocities of 0.16 and 0.3 m/s for air–water and air–Tellus oil systems. (a) and (b): Comparison of literature correlations for the centre-line velocity $V_L(0)$ for air–water system with Eulerian simulations. The plotted correlations are the same as in the legend to Fig. 1 c and d: comparison of Riquarts [18] correlation for the center-line velocity $V_L(0)$ for air–Tellus oil system with Eulerian simulations. Animations of the column start-up dynamics for the simulations can be viewed on our web site <http://ct-cr4.chem.uva.nl/oil-water>.

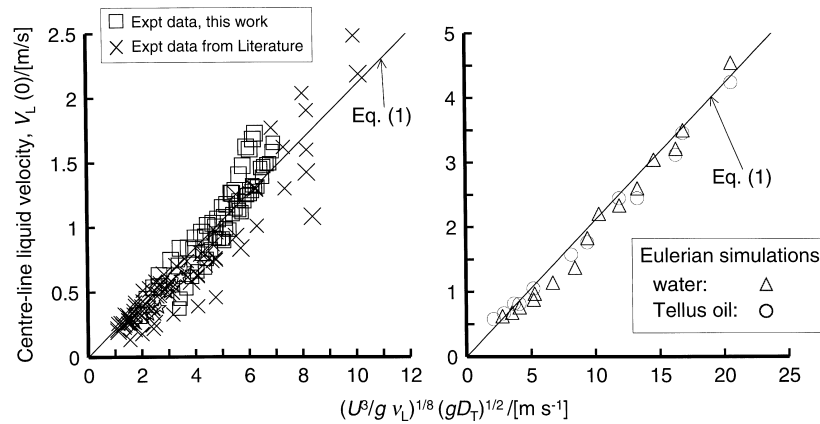


Fig. 8. (a) Comparison of experimental data for $V_L(0)$ obtained in this work for 0.174, 0.38 and 0.63 m diameter columns with literature data. The literature data were culled from Ueyama and Miyauchi ([17]; $D_T=0.6$ m), Zehner ([15]; $D_T=0.14$ m), Nottenkämper et al. ([19]; $D_T=0.45, 1.0$ m), Bernemann ([22]; $D_T=0.19, 0.45$ m), Hills ([24]; $D_T=0.14$ m), Pavlov (data quoted in [24] $D_T=0.7$ m) and Pozin (data quoted in [24]; $D_T=0.3$ m). (b) Comparison of Riquarts correlation (1) with Eulerian simulations for air–water and air–Tellus oil, for which details are given in Table 1. We take $\nu_L=10^{-6}$ m²/s for both water and Tellus oil, when applying Eq. (1).

set in the three columns the standard deviations of the literature correlations $\sqrt{\sum_i (V_L(0)_{\text{expt}} - V_L(0)_{\text{correlation}})^2 / N}$ are reported in Table 2 for the three columns. The smallest standard deviation was obtained with the Riquarts [18] correlation

$$V_L(0) = 0.21(gD_T)^{1/2} \left(\frac{U^3}{g\nu_L} \right)^{1/8} \quad (1)$$

The Riquarts correlation (1) anticipates a dependence of $V_L(0)$ on the kinematic viscosity of the liquid phase, ν_L . In order to test the validity of this viscosity dependence we compare the centre-line velocity measurements for water ($\nu_L=10^{-6}$ m²/s) and Tellus oil ($\nu_L=87 \times 10^{-6}$ m²/s) in Fig. 6a. Our experiments show only a small difference in the values of $V_L(0)$ with increasing liquid viscosity. The Riquarts correlation (1) works equally well for water and Tellus oil, if we take the properties corresponding to water, i.e. $\nu_L=10^{-6}$ m²/s for both systems. The Eulerian simulations also confirm that the liquid viscosity has a negligible influence on $V_L(0)$; see Fig. 6b.

For superficial gas velocities of 0.16 and 0.3 m/s, Eulerian simulations were carried out to study the influence of column diameter. In all simulations the column configurations and initial liquid heights were chosen such that the ratio of the dispersion height to the column diameter was about five; see Table 1. The results for air–water are shown in Fig. 7a and b. The Riquarts correlation again appears to be superior to the other literature correlations. Fig. 7c and d shows the corresponding simulation results for air–Tellus oil. These simulation results agree very well with the Riquarts correlation, taking properties corresponding to water, i.e., $\nu_L=10^{-6}$ m²/s.

Fig. 8a compares the Riquarts correlation (1) with the experimental data generated in this work and also data culled from the literature for columns of diameters ranging from

0.14 to 1.0 m. Though there is considerable spread in the literature data, it is clear that the Riquarts correlation provides a good estimation. In Fig. 8b we compare Eq. (1) with the results of Eulerian simulations. The agreement is excellent,

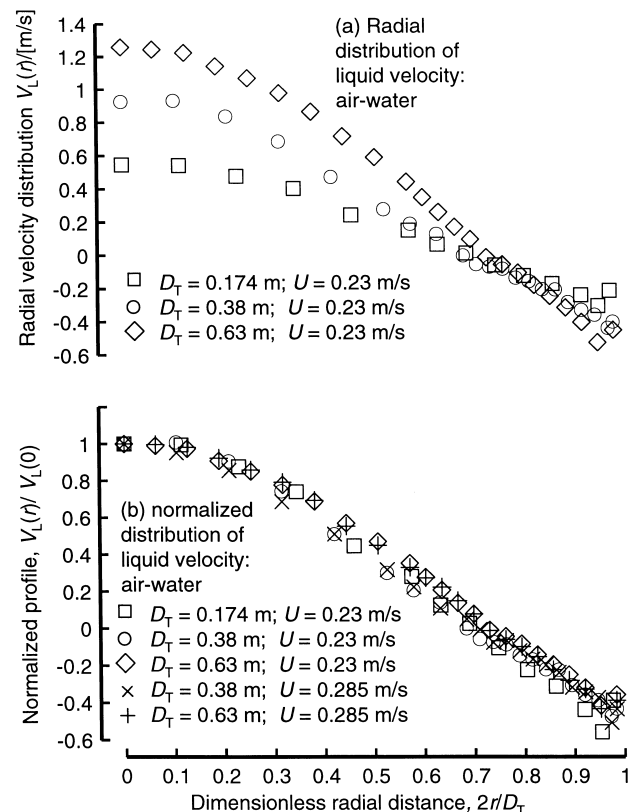


Fig. 9. (a) Radial distribution of the axial component of the liquid velocity at a superficial gas velocity $U=0.23$ m/s for three column diameters with the air–water system. (b) Normalized radial velocity distribution profiles for air–water system.

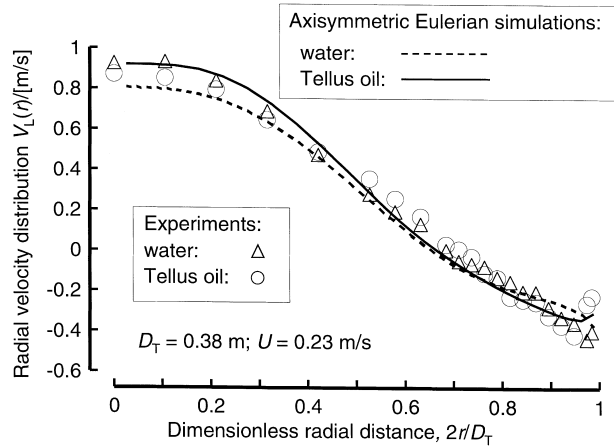


Fig. 10. Radial distribution of the axial component of the liquid velocity at superficial gas velocity $U=0.23$ m/s for air–water and air–Tellus oil system. Comparison of measurements in the 0.38 m diameter column with Eulerian simulations. Details for the Eulerian simulations can be found on the our web site: <http://ct-cr4.chem.uva.nl/oil-water>.

underlining the applicability of the Riquarts correlation for scale up to large diameter columns.

2.3. Radial distribution of liquid velocity $V_L(r)$

Fig. 9a shows the measured radial liquid velocity distribution $V_L(r)$ for the three columns operating at a superficial gas velocity of 0.23 m/s with the air–water system. The strong influence of the column diameter is evident. We note the strong downwardly directed liquid velocity in the wall region and the upwardly directed velocity in the central core. This liquid re-circulation is the cause of liquid phase dispersion and backmixing. If all the measured $V_L(r)$ data for air–water systems are normalized with respect to the centre line velocity, $V_L(0)$, we see that the radial distributions are all similar; see Fig. 9b. The important conclusion to be drawn from the result in Fig. 9 is that the magnitude of re-circulatory flows can be characterised by a single velocity, say $V_L(0)$. This would suggest that the liquid phase dispersion coefficient

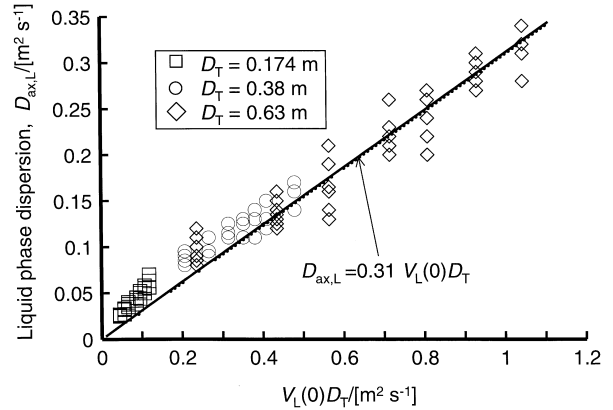


Fig. 12. Axial dispersion coefficient of the liquid phase. Comparison of measurements with our correlation $D_{ax,L}=0.31 V_L(0) D_T$. The standard deviation for this correlation= 0.02 m²/s.

$D_{ax,L}$ should be proportional to $V_L(0)$; we return to this point later.

For the 0.38 m diameter column, the radial distribution of the liquid velocity $V_L(r)$ measured with water and Tellus oil as the liquid phases, are compared in Fig. 10. The liquid viscosity does not have any significant influence on $V_L(r)$. This conclusion is also underlined by the Eulerian simulations of $V_L(r)$ shown in Fig. 10. We should also, therefore, expect no significant influence of liquid viscosity on the axial dispersion coefficient $D_{ax,L}$.

2.4. Liquid phase axial dispersion coefficient $D_{ax,L}$

In Fig. 11 we compare the experimental data on $D_{ax,L}$ with some typical literature correlations [8,9,11,12,15]. For the complete experimental data set in the three columns the standard deviations of the literature correlations $\sqrt{\sum_i (D_{ax,L}(0)_{\text{expt}} - D_{ax,L}(0)_{\text{correlation}})^2 / N}$, where N is the total number of experimental points=92, are as follows: Towell and Ackermann [12]=0.04 m²/s; Deckwer [9]=0.034; Baird and Rice [8]=0.032; Joshi [11]=0.035; Zehner [15]=0.035.

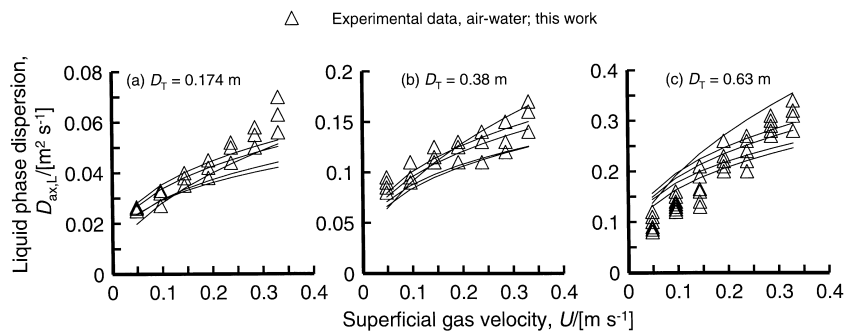


Fig. 11. Axial dispersion coefficient of the liquid phase. Comparison of measurements with literature correlations.

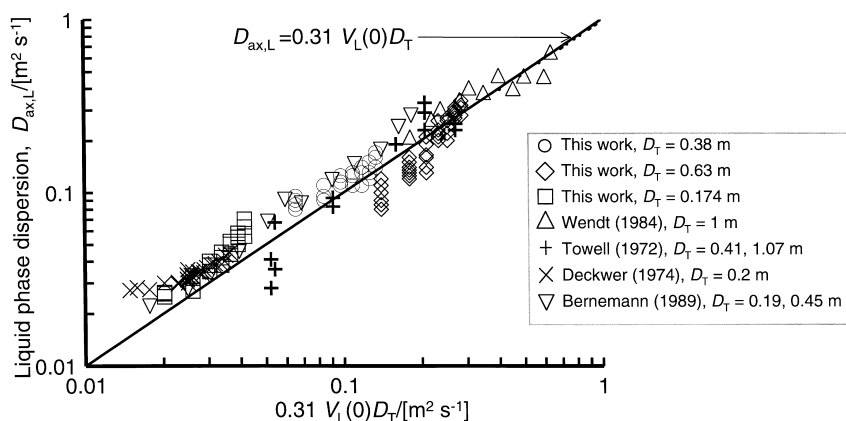


Fig. 13. Axial dispersion coefficient of the liquid phase. Comparison of experimental data obtained in this work and from the literature with our correlation $D_{ax,L} = 0.31 V_L(0) D_T$, wherein the centre-line liquid velocity is estimated from the Riquarts correlations (1) taking $V_L = 10^{-6} \text{ m}^2/\text{s}$.

For each experimental point, we also have measurements of the centre-line liquid velocity $V_L(0)$ and in Fig. 12 we plot the experimental values of $D_{ax,L}$ against the product of $V_L(0)$ and the column diameter. A simple linear proportionality is observed with a coefficient 0.31, i.e.,

$$D_{ax,L} = 0.31 V_L(0) D_T \quad (2)$$

The form of Eq. (2) was first suggested by Field and Davidson [10] and Joshi [11] and can be derived by considering dispersion in single phase turbulent flow. The standard deviation of Eq. (2) is $0.02 \text{ m}^2/\text{s}$, significantly lower than those of the other literature correlations. Since we have established that $V_L(0)$ is not significantly influenced by the liquid viscosity, we expect Eq. (2) to be valid for all systems. Following the conclusion reached earlier in this work, we recommend the use of the Riquarts correlation (1), with water properties, for estimation of the centre-line liquid velocity $V_L(0)$ for use in Eq. (2). This estimation procedure works very well not only for our own experimental data but also for data culled from the literature [9,12,14,22]; see Fig. 13.

3. Conclusions

The following major conclusions can be drawn from the present work.

1. The Riquarts correlation, Eq. (1), gives a reasonable prediction of the centre-line liquid velocity $V_L(0)$ in bubble columns operating in the churn-turbulent regime.
2. The liquid viscosity has a negligible influence on $V_L(0)$ and we recommend use Eq. (1) with properties of water, i.e., $\nu_L = 10^{-6} \text{ m}^2/\text{s}$. This conclusion is underlined also by Eulerian simulations.
3. Eulerian simulations have been used to verify the applicability of the Riquarts correlations to diameters up to 6 m.
4. The radial liquid velocity distribution profiles $V_L(r)$ are similar in shape for various column diameters and so the liquid circulations can be uniquely related to the

centre-line liquid velocity $V_L(0)$. The velocity profile $V_L(r)$ is not influenced by liquid viscosity, a conclusion also drawn from Eulerian simulations.

5. The liquid phase axial dispersion coefficient is best estimated from Eq. (2) wherein we use the Riquarts correlation (1) for $V_L(0)$.

4. Notation

$D_{ax,L}$	liquid phase axial dispersion coefficient (m^2/s)
D_T	column diameter (m)
g	acceleration due to gravity (9.81 m/s^2)
n	index of power-law dependence, dimensionless
N	number of experimental points, dimensionless
r	radial co-ordinate (m)
U	superficial gas velocity (m/s)
$V_L(r)$	radial distribution of liquid velocity (m/s)
$V_L(0)$	centre-line liquid velocity (m/s)

Greek letters

μ	viscosity of phase (Pa s)
ν	kinematic viscosity of phase (m^2/s)
ρ	density of phases (kg/m^3)
σ	surface tension of liquid phase (N/m)

Subscripts

expt	experimental
correlation	correlation
G	referring to gas phase
i	index
L	referring to liquid phase
T	tower or column

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