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MULTICOMPONENT MASS TRANSFER IN TURBULENT FLOW

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ABSTRACT

This paper analyses steady-state mass transfer between a wall, or interface, and a turbulently flowing fluid phase. The system is assumed to be multicomponent (number of species $\underline{n} > 2$) and proper account is taken of the diffusional coupling between the species transfers. The analysis results in a generalized procedure for the calculation of the elements of the matrix of multicomponent mass transfer coefficients from information on the eddy diffusivity in the boundary layer region, together with other physical and flow parameters. Appropriate correction factors to take account of high transfer rates have been defined and explicit expressions derived for these. Various special cases (small transfer rates, binary mass transfer, multicomponent Reynolds analogy) have been pointed out.

INTRODUCTION

The calculation of the mass transfer between a wall, or interface, and a turbulently flowing fluid phase is important in many process applications; examples include evaporation of lakes and rivers, condensation and evaporation of mixtures in annular flow inside tubes and transfer in falling films. Though most practical systems are multicomponent (here we define a multicomponent system in which the number of components exceeds two, i.e. $\underline{n} > 2$), most published treatments of the subject deal with binary or two-component systems ($\underline{n} = 2$). Now, multicomponent systems often display characteristics completely different from a two component system. Thus in a multicomponent system the transfer rate of any component is dependent on the transfer rates

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of all of the components in the mixture, i.e. we have to take account of interactions, or couplings, between the species transfers. There are some interesting consequences of multicomponent mass transfer interactions. It is possible to experience the phenomena of reverse diffusion (a component moves opposite to the direction dictated by its constituent driving force); osmotic diffusion (a component diffuses even though there is no driving present for its tranfer), diffusion barrier (a component fails to transfer even though its constituent driving force is non-zero). Such interaction phenomena have been experimentally observed in a few operations of chemical engineering interest; the reader is referred to the recent review of Krishna and Standart [1] for further information.

One important problem in the quantitative description of multicomponent mass transfer processes is the prediction, or calculation, of the matrix of multicomponent mass transfer coefficients, $[k^\bullet]$. For forced convection mass transfer, procedures for estimating $[k^\bullet]$ from binary correlations are available (see reference [1], for example). There are many situations of practical importance where the mass transfer takes place in a system of well defined geometry where detailed experimental information is available on the velocity distributions and fluctuations. In such cases the turbulent eddy diffusivity for mass transfer can be estimated. This information, together with the value of the molecular diffusivity, should in principle enable the calculation of the transfer coefficients and transfer rates. The recent excellent chapter by Sideman and Pinczewski [2] reviews the published treatments on binary mass transfer. The objective of the present communication is to extend the binary treatment to multicomponent systems. The literature on multicomponent mass transfer in turbulent flow is sparse [3 - 7; note the error in the multicomponent analysis of [3], as is pointed out by Stewart [5]]; the analysis presented in the present communication has not been carried out.

In the following analysis mass units and mass average reference velocity frames are consistently used throughout. For compactness of presentation all the symbols used in the text are defined fully in the nomenclature section and not in the main text. Compact matrix notation is used throughout. For an n-component system, treated here, the matrices are all $n-1$ dimensional; [ ] is used to represent an $n-1$ x $n-1$ dimensional square matrix; ( ) is used to denote a $n-1$ dimensional column matrix (vector). Clearly, for a binary system (n = 2) all the matrices reduce to one-dimensional, scalar, quantities.

ANALYSIS

We consider mass transfer between a wall, or an interface, denoted by subscript w, and a turbulently flowing fluid phase containing n components. The analysis covers flows in circular pipes and over flat plates specifically though extensions to other geometries and flow configurations are easily carried out. We assume a fully developed velocity profile in the flow direction (z-direction) and further that the mass transfer process is truly one dimensional (y-direction). The mass flux n_i for any component is considered positive if directed from the wall (subscript w) to the bulk fluid phase (subscript b). The mass flux n_i, referred to a stationary coordinate reference frame, is commonly split up into a purely diffusive contribution, j_{iy}, with respect to the mass average reference velocity, and a bulk flow, or convective contribution $\omega_i\, n_t$. If no chemical reaction takes place in the boundary layer considered then the mass fluxes n_i for component i and for the total mixture n_t are y-invariant for the flat plate geometry; for tube flows where the thickness of the viscous sub-layer and buffer layer are very small compared to the radius of the tube the assumption of constant n_i is a good approximation and we may write for both flow situations:

$$n_i = j_{iy} + \omega_i\, n_t = n_{iw} = n_{ib}; \quad i = 1,2,..n. \tag{1}$$

The mass diffusion flux j_{iy} is best expressed in terms of a generalized Fick's law formulation (see Stewart [5] and Standart and Krishna [7]):

$$(j_y) = -\rho\, [\, [D] + D^t\, [I]\,]\, \frac{d(\omega)}{d\, y}\,, \tag{2}$$

where the mass diffusion flux is seen to arise out of both molecular diffusion process and due to turbulent velocity fluctuations.

In the conventional way let us define the friction velocity u^*

$$u^* = (\tau_w/\rho)^{\frac{1}{2}}\,. \tag{3}$$

If we further take $\tau_w = \frac{1}{2}\, f \rho\, u_b^2$ then we see that the friction velocity can be written as:

$$u^* = (f/2)^{\frac{1}{2}}\; u_b\,. \tag{4}$$

A dimensionless distance parameter y^+ may be defined as:

$$y^+ = y\, u^*/\nu, \tag{5}$$

and the equation (1) written in terms of y^+:

$$(Z) = -\frac{\rho\, u^*}{n_t}\, [\, [Sc]^{-1} + Sc^{t\; -1}\, \frac{\nu^t}{\nu}\, [I]\,]\, \frac{d(\omega)}{d\, y^+} + (\omega) = (Z_w) = (Z_b) \tag{6}$$

where $Z_i = n_i/n_t$, the ratio of fluxes is also a y-invariant quantity. In the analysis to be presented in this paper we assume that the matrix of diffusion coefficients [D] is independent of composition and is thus y-invariant. The turbulent eddy diffusivity D^t is of course a strong function of y (and therefore of y^+) and assumes a value $D^t = 0$ at $y^+ = 0$. In proceeding further it is convenient to define a dimensionless matrix of mass transfer rate factors

$$[\Phi] = \frac{n_t}{\rho\, u^*} \left[[Sc]^{-1} + Sc^{t\ -1} \frac{\nu^t}{\nu} [I] \right]^{-1} \tag{7}$$

and an integral matrix $[F(y^+)]$

$$[F(y^+)] = \int_0^{y^+} [\Phi]\, dy^+ \tag{8}$$

which integral assumes a value $[F(y_b^+)]$ at the position $y^+ = y_b^+$ corresponding to the position y_b^+ where the composition (mass fraction) ω_i reaches the bulk fluid stream value ω_{ib}.

With the definitions (7) and (8) above it is easy to solve the matrix differential equation (6) to yield the composition profile in the form:

$$(\omega - \omega_w) = [\ \exp[F(y^+)] - [I]\ ]\ [\ \exp[F(y_b^+)] - [I]\ ]^{-1} (\omega_b - \omega_w) \tag{9}$$

The composition profile given by equation (9) can be differentiated to yield the mass fraction gradient at $y^+ = 0$:

$$\left.\frac{d(\omega)}{d\,y^+}\right|_{y^+=0} = -[\Phi]\ [\ \exp[F(y_b^+)] - [I]\ ]^{-1}\ (\omega_w - \omega_b). \tag{10}$$

Combining equations (1) - (10) we can write the diffusion flux at the wall ($y^+ = 0$), j_{iw}, as:

$$(j_w) = n_t\ [\ \exp[F(y_b^+)] - [I]\ ]^{-1}\ (\omega_w - \omega_b) \tag{11}$$

and if we <u>define</u> a matrix of mass transfer coefficients $[k_w^\bullet]$ by

$$(j_w) \equiv \rho\ [k_w^\bullet]\ (\omega_w - \omega_b) \tag{12}$$

then we obtain:

$$[k_w^\bullet] = n_t\ [\ \exp[F(y_b^+)] - [I]\ ]^{-1} / \rho\ . \tag{13}$$

Equation (13) represents a major new result of this communication and before we can rewrite this relation in a more usable form it is necessary to consider the special case in which we have vanishingly small transfer fluxes (n_i, i=1,2,..$\underline{n}$, all tending to vanish and consequently the total flux n_t also tends to vanish: $n_t = 0$); we call this the small mass transfer case.

SMALL MASS TRANSFER RATES

In the limit of vanishingly small fluxes we have $n_i = j_i$ and there is no need to make a distinction between $[k_w]$ and $[k_b]$; we have a single matrix of mass transfer coefficients [k], which can be determined by taking the limiting case of equation (13) as n_t tends to zero. Thus we obtain

$$[k]^{-1} = [G(y_b^+)]/u^* \tag{14}$$

where the matrix function $[G(y_b^+)]$ is given by

$$[G(y_b^+)] = \int_0^{y_b^+} [\,[Sc]^{-1} + Sc^{t\,-1}\frac{\nu^t}{\nu}[I]\,]^{-1}\,dy^+ . \tag{15}$$

If we further define a matrix of Stanton numbers by the relation:

$$[St] = [k]/u_b \tag{16}$$

then it is easy to derive the following expression for $[St]^{-1}$:

$$[St]^{-1} = (2/f)^{\frac{1}{2}} \quad [G(y_b^+)] . \tag{17}$$

One difficulty is that the position y_b^+, where the mass fraction ω_i reaches the bulk fluid stream value ω_{ib}, is not known. In proceeding to evaluate the integral $[G(y_b^+)]$ using equation (15) it is convenient to split the integral in two parts (see the corresponding binary treatment in Sideman and Pinczewski [2] and Sherwood [8,9]):

1): $0 - y_1^+$ in which the turbulent diffusivities and molecular diffusivities are both important in determining the system transport behaviour

2): $y_1^+ - y_b^+$ in which the turbulent diffusivities predominate over the molecular diffusivies ($\nu^t >> \nu$).

In proceeding further in the analysis we assume that $Sc^t = 1$ as is commonly done in turbulent heat and mass transfer analyses. Splitting the integral as indicated above we may write for $[St]^{-1}$:

$$[St]^{-1} = \left(\frac{2}{f}\right)^{\frac{1}{2}} \int_0^{y_1^+} [\,[Sc]^{-1} + \frac{\nu^t}{\nu}\,[I]\,]^{-1}\,dy^+ \quad +$$

$$+ \left(\frac{2}{f}\right)^{\frac{1}{2}} \int_{y_1^+}^{y_b^+} [\frac{\nu^t}{\nu}\,[I]\,]^{-1}\,dy^+ . \tag{18}$$

Now the shear stress at any position y^+ within the boundary layer is

$$\frac{\tau}{\tau_w} = (1 + \frac{\nu^t}{\nu})\,\frac{du^+}{dy^+} = 1 \tag{19}$$

and the second integral on the right hand side of equation (18) may be written as

$$\int_{y_1^+}^{y_b^+} [\,\frac{\nu^t}{\nu}\,[I]\,]^{-1}\,dy^+ = u_b^+\,[I] - \int_0^{y_1^+} [\,[I] + \frac{\nu^t}{\nu}\,[I]\,]^{-1}\,dy^+ \tag{20}$$

where $u_b^+ = u_b/u^* = (2/f)^{\frac{1}{2}}$. With the help of equation (20) we obtain:

$$[St]^{-1} = \frac{2}{f}\,[I] + \left(\frac{2}{f}\right)^{\frac{1}{2}} \int_0^{y_1^+} \left[\,[[Sc]^{-1} + \frac{\nu^t}{\nu}\,[I]]^{-1} - [[I] + \frac{\nu^t}{\nu}\,[I]]^{-1}\right] dy^+ \tag{21}$$

which relation allows us to evaluate $[k]$ provided y_1^+ is specified. There are many eddy diffusivity models available in the literature [2,8,9] and there are also many corresponding choices for the distance y_1^+. For a a discussion on the choice of y_1^+ see Sherwood [8].

The matrix of mass transfer coefficients [k] calculated with the help of equation (21) are those obtained under conditions of vanishingly small mass transfer fluxes; in the chemical engineering literature these are referred to as zero-flux mass transfer coefficients [1]. Once an appropriate relation is available for ν^t as a function of y^+, the integral in eq. (21) can be evaluated. Any model may be used for this purpose, e.g. Deissler, Taylor-Prandtl, von Kármán, Martinelli, etc. [2,8,9].

Now there are many applications in practice where the mass transfer fluxes are not small. One practical example is the evaporation of a mixture of liquids with high volatility. In such cases the complete relation (13) must be used. For $n_t = 0$, $[k] = [St]\, u_b$ where [St] is given by eq. (21).

TRANSFER COEFFICIENTS FOR FINITE TRANSFER RATES

Below we suggest a procedure for the calculation of the matrix of finite flux mass transfer coefficients $[k_w^\bullet]$.

It is easy to see from equations (7), (8), (15) - (17) that:

$$[F(y_b^+)] \equiv \frac{n_t}{\rho}\,[k]^{-1} \tag{22}$$

where the evaluation of the right hand side has been discussed in the previous section. With the aid of equation (22) we may write equation (13) in the form:

$$[k_w^\bullet] = [k]\,[\Xi]; \quad [\Xi] \equiv [F(y_b^+)]\,[\exp[F(y_b^+)] - [I]]^{-1} \tag{23}$$

where $[\Xi]$ is a matrix of correction factors to account for high mass transfer rates. It can be easily checked that the correction factor matrix $[\Xi]$ reduces to the identity matrix [I] for conditions of small transfer rates ($n_t = 0$). Clearly a trial and error procedure is required for the calculation of the matrix of finite flux mass transfer coefficients $[k_w^\bullet]$. We present below a step-by-step procedure for the calculation of the matrix $[k_w^\bullet]$ and of the transfer fluxes n_i from a knowledge of the wall compositions (ω_w) and the bulk flow variables (ω_b, u_b).

Step 1: Assume a certain turbulent eddy diffusivity functional relationship (i.e. $\nu^t = \nu^t(y^+)$); such relations can be found for example in [2].

Step 2: Estimate the matrix of molecular diffusion coefficients [D]; such estimation procedures are discussed in [1].

Step 3: From the information is Steps 1 and 2 and from a knowledge of the physical properties of the fluid (density, viscosity), the flow hydrodynamics (u_b, friction factor f, etc), the matrix of Stanton numbers can be evaluated by evaluating the integral (analytically or numerically) in equation (21).

Step 4: The zero-flux matrix of mass transfer coefficients [k] can then be calculated by using equation (16).

Step 5: With the estimate of the matrix of zero-flux transfer coefficients the first estimate of the matrix of finite flux mass transfer coefficients can be obtained by using the relation:

$$[k_w^\bullet] = [k]\,[\Xi] \tag{24}$$

where for the first iteration we assume that $[\Xi] = [I]$, the identity matrix.

Step 6: The diffusion fluxes (j_w) can be calculated using equation (12).

Step 7: From the knowledge of (j_w) we can calculate the fluxes in a stationary coordinate reference frame, n_i, provided an additional piece of information is available (see [1] for a discussion on this point). Thus if we have diffusion of n-1 components in the presence of a non-transferring, or inert component, n, then $n_n = 0$. This is the case when we have evaporation of volatile components in air, for example. A relation such as $n_n = 0$ is sufficient to allow the calculation of the n fluxes n_i from the n-1 independent mass diffusion fluxes at the wall, j_{iw}. Thus for $n_n = 0$:

$$(n) = [\beta](j_w) \tag{25}$$

where the matrix $[\beta]$, termed the bootstrap matrix [1], has the elements given by:

$$\beta_{ik} = \delta_{ik} + \omega_{iw}/\omega_{nw}, \qquad i,k = 1,2,\ldots n-1 \tag{26}$$

Step 8: From the estimate of the fluxes n_i from Step 7 we can calculate n_t and therefore the elements of $[F(y_b^+)]$ from equation (22). The matrix of correction factors $[\Xi]$ can then be calculated from equation (23).

Step 9: From the last estimate of $[\Xi]$, we may repeat Steps 5 - 8 till we obtain convergence on each individual flux n_i.

BINARY MASS TRANSFER AS A SPECIAL CASE OF GENERAL ANALYSIS

The above analysis for the case of n-component transfer can of course be applied to the case n = 2, discussed in a great variety of texts. Our analysis above has been developed for the case of finite mass transfer rates and to the knowledge of the author even this case has not been tackled in the literature. Therefore it is worthy of attention.

For a binary system the Stanton number is given by:

$$\frac{1}{St} = \frac{2}{f} + \left(\frac{2}{f}\right)^{\frac{1}{2}} \int_0^{y_b^+} \left[\frac{1}{1/Sc + \nu^t/\nu} - \frac{1}{1 + \nu^t/\nu} \right] dy^+ \tag{27}$$

which for Sc = 1 reduces to

$$St = f/2 \tag{28}$$

which is Reynolds analogy. The zero-flux mass transfer coefficient is given by $k = St\, u_b$. The finite flux mass transfer coefficient $k_w^\bullet$, at the wall, is given by the binary analog of eq. (24):

$$k_w^{\bullet} = k\,\Xi \tag{29}$$

where the correction factor Ξ for finite fluxes is given by

$$\Xi \equiv \frac{F(y_b^+)}{\exp(F(y_b^+)) - 1} . \tag{30}$$

It is again easy to check that the correction factor given by eq. (30) reduces to unity for vanishing n_t; when this is the case we have $k_w^{\bullet} = k$. This is the situation considered in most treatments of binary mass transfer [2]. In evaluating $F(y_b^+)$ we need to recall that (compare with eq. (22))

$$F(y_b^+) \equiv n_t/\rho\, k \tag{31}$$

MULTICOMPONENT REYNOLDS ANALOGY

For a binary system St = f/2 when we have the special situation that the molecular Schmidt number equals unity (Sc = 1); this situation can be realized for an ideal gas mixture of two-components. For a multicomponent system, in general, there will be differences between the constituent binary pair diffusivities and if one constituent binary pair Schmidt number equals unity, it is unlikely that the same will hold for other binary pairs. In other words, the equivalent situation [Sc] = [I] for a multicomponent system is a much more special case than a binary system. Nevertheless, it is interesting to present formally the multicomponent analog of Reynolds analogy as follows (this follows from eq. (21)):

$$[St] = \frac{f}{2}[I] \quad \text{if} \quad [Sc] = [I]. \tag{32}$$

TURBULENT DIFFUSIVITY MODELS

The analysis presented so far in this paper allows the calculation of the mass transfer coefficients $[k_w^{\bullet}]$ provided an expression is available for for the eddy diffusivity of momentum ν^t. Models for ν^t are discussed widely in the literature (see for example [2] and [8]). We would like to give here one such expression to illustrate the procedure. If we choose von Kármán's model we can derive (see Sherwood [8,9] for details for binary systems):

$$[St]^{-1} = \frac{2}{f}[I] + 5\left(\frac{2}{f}\right)^{\frac{1}{2}}\left[\,[\,Sc - I\,] + \ln\left[[I] + \frac{5}{6}[\,Sc - I\,]\right]\right]. \tag{33}$$

In deriving eq. (33) from eq. (21) we have assumed $y_1^+ = 30$ together with the von Kármán universal velocity profile. Other published binary models [8,9] can be similarly generalized to multicomponent systems.

CONCLUDING REMARKS

We have considered mass transfer between a wall, or interface, and a flowing bulk fluid stream. For the case in which the hydrodynamics of the flow in the region of the boundary layer is well-established, i.e. the velocity profiles in the region of the boundary layer are known, the parameter characterising the eddy transport due to turbulence (ν^t, D^t) can be estimated. The equations of continuity describing multicomponent mass transfer are cast into n-1 dimensional matrix notation and an expression obtained for the calculation of the mass transfer coefficients in terms of known quantities. The contribution of the work presented here is that the usual treatments of mass transfer have been generalized in two respects:

1) conventional treatments have been extended to multicomponent systems while taking into account the molecular diffusional couplings between the diffusing species ([D] has non-zero off-diagonal elements)
2) conventional treatments have been extended to take account of high rates of transfer from the wall to the fluid. In this respect it is appropriate to mention that the expressions (23) and (30) represent the analogs of the well known Ackermann correction factor accounting for the effect of mass transfer on condensation heat transfer [1].

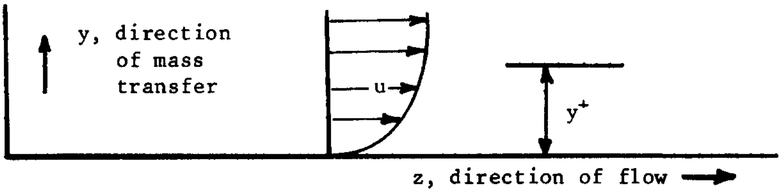
The calculation procedure for the case of high transfer rates involves a trial-and-error procedure but the head-to-tail iteration procedure suggested has been found to be fast in practical examples tested.

The analysis presented here has many applications, for example:

1) calculation of transfer rates in the gas phase during condensation or evaporation with a tube; the procedure given above for the estimation of mass transfer coefficients can be incorporated into the design procedure suggested by Krishna et al [10,11]
2) calculation of evaporation rates of multicomponent liquid mixtures flowing inside tubes (annular flow)
3) calculation of the mass transfer resistances encountered during reaction of gaseous components flowing in tubes and the tube wall coated with catalyst or solid reactant (e.g. catalytic mufflers).

The analysis presented in this article can be extended to the case of coupled heat and multicomponent mass transfer where thermal diffusion and Soret effects can be important.

NOMENCLATURE



(note: we consider only fully developed flow in this paper)

D binary molecular diffusion coefficient; (m^2 s^{-1})

[D] matrix of molecular diffusion coefficients, n-1 x n-1 dimensional; assumed independent of composition and position; (m^2 s^{-1})

D^t turbulent eddy diffusivity of mass; (note that in turbulent mass mass transport there are no turbulent diffusional couplings; the matrix $[D^t]$ reduces to a scalar times the identity matrix: $[D^t] = D^t [I]$); (m^2 s^{-1})

f Fanning friction factor; (dimensionless)

$[F(y^+)]$ integral defined by equation (7),(dimensionless)

$[G(y^+)]$ integral defined by equaiton (15),(dimensionless)

[I] identity matrix with elements given by δ_{ik} (dimensionless)

j_{iy} mass diffusion flux of component i relative to the mass average reference velocity; $j_{iy} = n_i - \omega_i n_t$; j_{iy} is y-position dependent (kg m^{-2} s^{-1})

(j_w) column matrix of mass diffusion fluxes evaluated at y = 0 (wall) (kg m^{-2} s^{-1})

[k] matrix of zero-flux multicomponent mass transfer coefficients; n-1 x n-1 dimensional; (m s^{-1})

$[k^\bullet]$ matrix of finite-flux multicomponent mass transfer coefficients; n-1 x n-1 dimensional; (m s^{-1})

k binary mass transfer coefficient; (m s^{-1})

$[k_w^\bullet]$ matrix of finite flux multicomponent mass transfer coefficients evaluated at the wall ($y^+ = 0$) (m s^{-1})

n number of components in fluid mixture (including inerts)

n_i mass flux of component i with respect to a stationary coordinate reference frame; (kg m^{-2} s^{-1})

n_t total mixture mass flux; $n_t = \sum_{i=1}^{n} n_i$; (kg m^{-2} s^{-1})

Sc Schmidt number for binary system; $Sc = \nu/D$ (dimensionless)

Sc^t turbulent Schmidt number; $Sc^t = \nu^t/D^t$; for a multicomponent system we have $[Sc^t] = Sc^t\ [I]$, i.e. no turbulent diffusional couplings; usually we take $Sc^t = 1$; Note that in the text $Sc^{t\ -1}$ denotes $1/Sc^t$; (dimensionless)

$[Sc]$ matrix of Schmidt numbers; $[Sc] = \nu\ [D]^{-1}$; (dimensionless)

St Stanton number for binary system; $St = k/u_b$; (note that this number is defined only for conditions of vanishingly small mass transfer fluxes); (dimensionless)

$[St]$ matrix of Stanton numbers defined by $[St] = [k]/u_b$; (defined only for conditions of vanishingly small transfer fluxes); (dimensionless)

u fluid velocity along the conduit or over a flat plate; ($m\ s^{-1}$)

u_b for flow in a circular pipe u_b is the volumetric flow rate of fluid divided by the cross-sectional area of pipe; for flow over flat plate u_b is to be taken as the bulk fluid stream velocity; ($m\ s^{-1}$)

u^* friction velocity defined by eq. (3) or (4) ($m\ s^{-1}$)

u^+ dimensionless velocity defined by $u^+ = u/u^*$ (dimensionless)

y position coordinate; distance from wall (m)

y^+ dimensionless distance coordinate from wall defined by eq.(5); (dimensionless)

y_1^+ distance from the wall within which both molecular and turbulent diffusivities play a role; (dimensionless)

y_b^+ distance from wall at which the mole fraction reaches the bulk stream value ω_{ib}; (dimensionless)

Z_i ratio of flux of i to total mixture flux; (dimensionless)

(Z) column matrix of flux ratios; (dimensionless)

<u>Greek Letters</u>

$[\beta]$ bootstrap solution matrix which allows the calculation of the fluxes n_i from knowledge of the mass diffusion fluxes j_i; elements of $[\beta]$ for Stefan diffusion are given by eq.(26); (dimensionless)

δ_{ik} Kronecker delta

μ fluid phase viscosity, assumed independent of position; (Pa s)

ν fluid kinematic (molecular) viscosity ($m^2\ s^{-1}$)

ν^t turbulent eddy kinematic viscosity, function of y^+; ($m^2\ s^{-1}$)

$[\Xi]$	matrix of correction factors to account for finite mass transfer rates defined by eq. (23); (dimensionless)
ρ	fluid mixture mass density; (kg m^{-3})
τ	shear stress; (N m^{-2})
$[\Phi]$	matrix of mass transfer rate factors defined by eq. (7); (dimensionless)
ω_i	mass fraction of component i; (dimensionless)
(ω)	column matrix of mass fractions; (dimensionless)
(ω_w)	column matrix of mass fractions at wall; (dimensionless)
(ω_b)	column matrix of mass fractions in bulk fluid; (dimensionless)

Matrix Notation

[ ]	n-1 x n-1 dimensional square matrix
$[\ ]^{-1}$	n-1 x n-1 inverted square matrix
( )	column matrix with n-1 elements

Subscripts

b	bulk fluid parameter
n	pertaining to component n
t	pertaining to total mixture
w	pertaining to wall

Superscripts

+	non-dimensionalized parameter
•	high-flux mass transfer entity
t	turbulent property
*	friction velocity

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