

# Kinetic Monte Carlo Simulations of the Loading Dependence of Diffusion in Zeolites

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Kinetic Monte Carlo (KMC) simulations are used to model diffusion of molecules in zeolites. A variety of loading dependences of the Maxwell-Stefan diffusivity,  $\mathcal{D}_i$ , can be realized by allowing the jump frequencies,  $\nu$ , of molecules to be influenced by the presence of neighboring molecules. Neighboring molecules are assumed to reduce or increase the activation energy for diffusion by  $\delta E$  and the jump frequencies  $\nu$  are altered by a factor  $f = \exp(\delta E/RT)$  for each neighboring molecule. By appropriate choice of  $\nu$  and  $f$  the loading dependence of  $\mathcal{D}_i$  can be made to match those obtained from either Molecular Dynamics (MD) simulations or experiment. The major advantage of the KMC simulation strategy is that considerably less computer power is required than for the corresponding MD simulations. Furthermore, KMC simulations allow mixture diffusion to be probed without additional parameter tuning. The KMC simulations also validate the applicability of the quasi-chemical theory of Reed and Ehrlich (*Surf. Sci.* **1981**, *105*, 603–628) for describing the loading dependence of  $\mathcal{D}_i$ .

## 1 Introduction

Zeolites are crystalline nanoporous materials that are widely used in the chemical industry as catalysts and adsorbents [1–3]. Zeolite crystals are incorporated into binders and used in the form of pellets in fixed or simulated moving bed reactors. Alternatively, zeolite crystals are coated on to a porous membrane support and used in (catalytic) membrane reactors and separation devices [4]. More than 100 different zeolite structures are encountered in practice [5]. Viewed simply, these structures fall into three broad categories, (a) structures with intersecting channels (examples include MFI, MEL, BOG), (b) cages separated by windows (LTA, FAU), and (c) essentially one-dimensional channels (AFI, MTW), sometimes with side pockets (MOR).

In the development of separation and reaction processes using zeolites, the proper description of diffusion of molecules within zeolites is an essential step and it is generally accepted that the Maxwell-Stefan (M-S) diffusion formulation provides a convenient and general framework that can be used in practice [6–8]. For single component diffusion of species  $i$ , the M-S formulation<sup>1)</sup>

$$N_i = -\rho\Theta_i\mathcal{D}_i\frac{1}{RT}\nabla\mu_i \equiv -\rho\Theta_{i,sat}\mathcal{D}_i\frac{\theta_i}{RT}\nabla\mu_i \quad (1)$$

relates the flux  $N_i$  to the chemical potential gradient  $\nabla\mu_i$ . In Eq. (1),  $\Theta_i$  is the molecular loading expressed say in molecules per unit cell,  $\Theta_{i,sat}$  is the saturation loading,  $\theta_i \equiv \Theta_i/\Theta_{i,sat}$  is the fractional occupancy,  $\rho$  is the zeolite framework density expressed as the number of unit cells per  $\text{m}^3$ ,  $R$

is the gas constant,  $T$  is the temperature, and  $\mathcal{D}_i$  is the M-S diffusivity.

Both Molecular Dynamics (MD) simulations [9,10] and experimental studies with MFI zeolite [11–14] have revealed a wide variety of dependencies of the Maxwell-Stefan diffusivity,  $\mathcal{D}_i$ , on the loading  $\Theta_i$ . The important advantage of MD simulations is that with the appropriate choice of the force field it is possible to obtain the correct estimate of both the magnitude of  $\mathcal{D}_i$  and its dependence on the loading  $\Theta_i$  [11,12]. However, MD simulations are computationally very demanding. Kinetic Monte Carlo (KMC) simulations [6,15] demand much less computer power but do not possess the predictive power of MD simulations. The jump frequencies of molecules,  $\nu_i$ , need to be supplied as inputs to KMC simulations; these can be estimated from the transition state theory [16] or MD simulations [6,15,17]. Using the standard KMC simulation strategy the occupancy dependence of  $\mathcal{D}_i$  invariably follows the dependence

$$\mathcal{D}_i = \mathcal{D}_i(0)(1 - \theta_i) \quad (2)$$

Eq. (2) was found to hold for example for 2-methylhexane (2MH) in MFI [17]. In order to reproduce the wide variety of observed loading dependences other than that portrayed by Eq. (2) it is necessary to allow the jump frequencies to be influenced by each of the neighboring molecules by a factor  $f = \exp(\frac{\delta E}{RT})$ , where  $\delta E$  represents the reduction in the energy barrier, following the KMC simulation strategy as described elsewhere [18–21] and the details are not repeated here.

The first major objective of the present communication is to show the KMC simulations can be properly *tuned* to reproduce either MD or experimental  $\mathcal{D}_i - \Theta_i$  data. Secondly, we also demonstrate that the KMC data verify the model of Reed and Ehrlich [22], appropriately generalized as follows [19]:

$$\mathcal{D}_i(\theta) = \mathcal{D}_i(0) \frac{(1+\varepsilon)^{z-1}}{(1+\varepsilon/f)^z} \quad (3)$$

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1) List of symbols at the end of the paper.

where  $z$  is the coordination number, representing the maximum number of nearest neighbors and the other parameters are given by

$$f = \exp\left(\frac{\delta E}{RT}\right); \quad \varepsilon = \frac{(\beta - 1 + 2\theta)f}{2(1 - \theta)};$$

$$\beta = \sqrt{1 - 4\theta(1 - \theta)(1 - 1/f)} \quad (4)$$

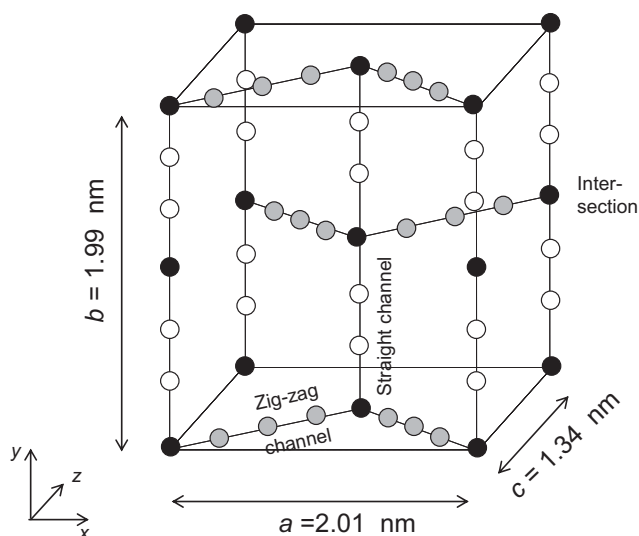
The Reed-Ehrlich model provides a convenient mean-field approach to modeling the occupancy dependence of  $D_i$  that can be used for engineering purposes.

The final objective is to demonstrate that if individual species in a mixture are tuned in this manner, KMC simulations can be used to probe the mixture diffusion characteristics with considerably less computer power than the corresponding MD simulations.

## 2 Single Component Diffusion in Various Topologies

Let us first consider diffusion in square and cubic lattices for which the coordination numbers are  $z = 4$  and  $6$ , respectively. The KMC simulations of the M-S diffusivity, normalized with respect to the zero-loading diffusivity,  $D_i/D_i(0)$  for square and cubic lattices are shown by open symbols in Figs. 1a, b for varying values of the factor  $f = 0.5, 1, 1.5$ , and  $2$ . Also shown with continuous solid lines in Figs. 1a, b are the corresponding calculations using the Reed-Ehrlich model, Eq. (3). The agreement between the KMC simulations and the Reed-Ehrlich model is perfect as expected, because the KMC simulations were set up using exactly the same physical model underlying the Reed-Ehrlich development.

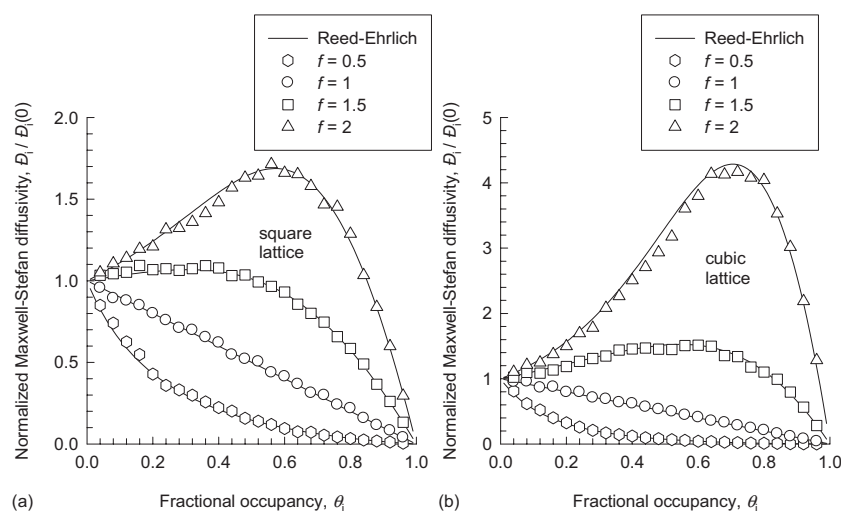
Now let us turn to diffusion of 2-methylhexane (2MH) in MFI zeolite at 300 K. From configurational considerations this branched alkane can only be located at the intersections between the straight (st) and zig-zag (zz) channels [23]; see



**Figure 2.** Unit cell for MFI lattice topology with a total of 24 sites per unit cell; these sites are indicated as black circles (intersections), white circles (straight channels) and grey circles (zig-zag channels). The distribution of the 24 molecules per unit cell is as follows: 4 at intersections, 8 within straight channels, and 12 within the zig-zag channels.

Fig. 2. The saturation capacity is therefore  $\Theta_{i,sat} = 4$  molecules per unit cell and so we take the total number of adsorption sites as 4, all located at the intersections. The jump frequencies of 2MH along the straight channels,  $\nu_{st}$ , and zig-zag channels,  $\nu_{zz}$ , can be estimated using the transition state theory [24] and are specified in Tab. 1. The KMC simulation results are shown by the open symbols in Fig. 3a for  $f = 0.5, 1, 1.5$ , and  $2$ . The continuous solid lines in Fig. 3a are the calculations using the Reed-Ehrlich model, Eq. (3), taking  $z = 4$ . The choice of the coordination number is dictated by the fact that each molecule at the intersections has four neighboring molecules, also located at the intersections.

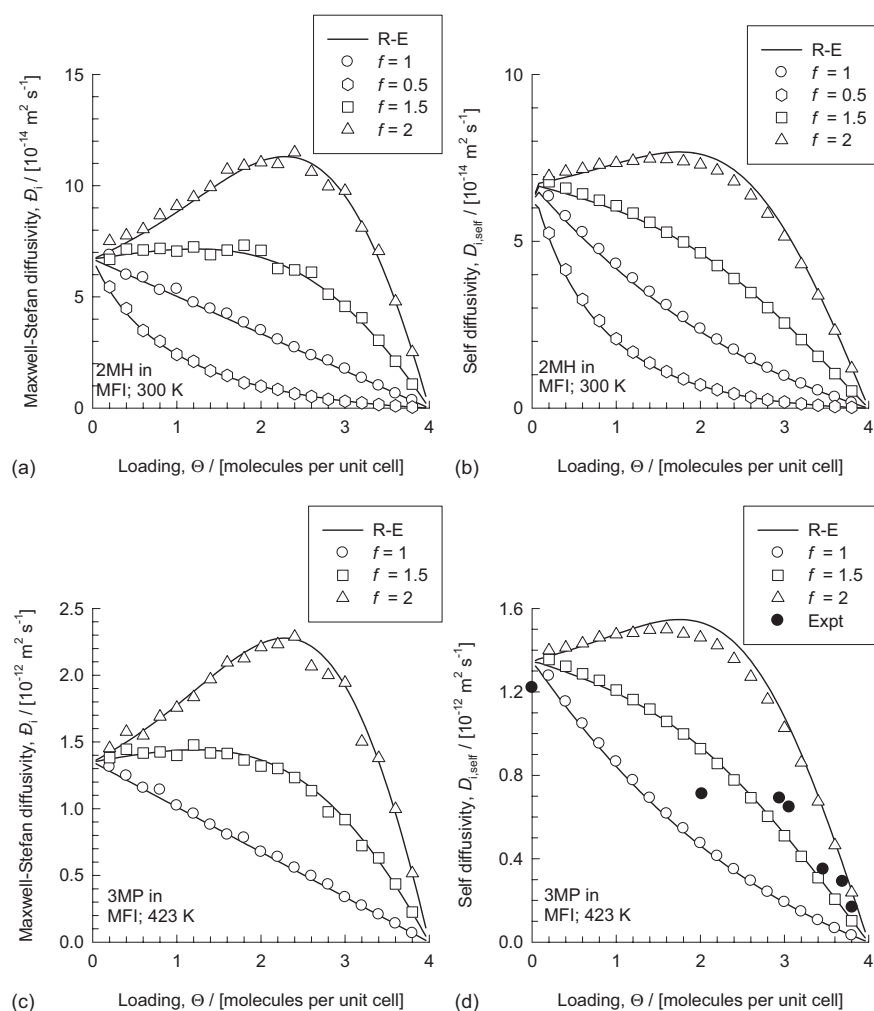
The corresponding KMC simulations of the *self*-diffusivity of 2MH in MFI are shown in Fig. 3a for  $f = 0.5, 1, 1.5$  and  $2$ . The continuous solid lines in Fig. 3a were obtained from



**Figure 1.** KMC simulations (open symbols) of normalized M-S diffusivity,  $D_i/D_i(0)$  in (a) square, and (b) cubic lattice topologies. Also shown using continuous solid lines are the calculations of  $D_i/D_i(0)$  with the Reed-Ehrlich model, Eq. (3).

**Table 1.** Model parameters for various molecules in MFI zeolite. The temperatures are in degrees K. The jump frequencies  $\nu_{st}$  and  $\nu_{zz}$  have the unit  $s^{-1}$ . The zero-loading M-S diffusivities  $D_i(0)$  have the units of  $m^2 s^{-1}$ . The other parameters  $a_1$  and  $a_2$  are dimensionless.

Molecule	Temperature	Number of sites	$\nu_{st}$	$\nu_{zz}$	$z$	$D_i(0)$	$D_{ii}/D_i = a_1 \exp(-a_2\theta)$	
							$a_1$	$a_2$
2MH	300	4	$1.4 \times 10^5$	$5 \times 10^4$	4	$6.7 \times 10^{-14}$	1.4	0.5
3MP	423	4	$2.8 \times 10^6$	$1 \times 10^6$	4	$1.35 \times 10^{-12}$	1.4	0.5
N <sub>2</sub>	200	20	$2.2 \times 10^{11}$	$1.1 \times 10^{11}$	2.4	$0.8 \times 10^{-8}$	0.7	1.2
CO <sub>2</sub>	300	24	$2 \times 10^{11}$	$1 \times 10^{11}$	2.33	$0.55 \times 10^{-8}$	0.5	0.9
CH <sub>4</sub>	298	20	$4 \times 10^{11}$	$2 \times 10^{11}$	2.4	$1.45 \times 10^{-8}$	0.7	1.2
CF <sub>4</sub>	298	20	$1.2 \times 10^{11}$	$7 \times 10^{10}$	2.4	$0.47 \times 10^{-8}$	0.7	1.2



**Figure 3.** KMC simulations (open symbols) of M-S diffusivity  $D_i$  and self diffusivity  $D_{i,self}$  in MFI of (a, b) 2MH at 300 K, and (c, d) 3MP at 423 K. Also shown using continuous solid lines are the calculations of  $D_i$  with the Reed-Ehrlich model, Eq. (3). The filled black circles in (d) represent the experimental values reported by Koriabkina *et al.* [13].

calculations using the following formula for the self-diffusivity,  $D_{i,self}$

$$D_{i,self} = \frac{1}{\frac{1}{D_i} + \frac{\theta_i}{D_{ii}}} \quad (5)$$

where  $D_{ii}$  is the self-exchange coefficient that arises naturally in the M-S formulation for tracer diffusion of a labeled species in an environment of unlabeled species [8]. The parameter  $D_{ii}/D_i$ , can be taken as a measure of correlation effects. The lower the value of  $D_{ii}/D_i$ , the stronger the influence of correlations. A very large value of  $D_{ii}/D_i$  signifies weak correlations between molecular jumps. The  $D_{ii}/D_i$  is also occupancy dependent and this dependence can be generally described by

$$\frac{D_{ii}}{D_i} = a_1 \exp(-a_2 \theta_i) \quad (6)$$

following the work of Skoulidas *et al.* [8]. For the 2MH diffusion in MFI the parameters  $a_1$  and  $a_2$  were chosen as 1.4 and 0.5. The good agreement between Eq. (5) and the KMC simulations for a variety of values of the interaction parameter implies that the self-exchange coefficient is *not* influenced by the chosen value for  $f$ . This is a useful simplification to the description of diffusion in mixtures, in which  $D_{ii}$  plays a crucial role [8].

Koriabkina *et al.* [13] report experimental data on the loading dependence of the self-diffusivity of 3-methylpentane (3MP) in MFI at 423 K. We try to rationalize the experimentally observed loading dependence by performing KMC simulations of both  $D_i$  and  $D_{i,self}$  for a variety of values of  $f$ ; the results are shown by the open symbols in Figs. 3c, d. In performing these simulations the ratio of the jump frequencies  $\nu_{st}/\nu_{zz}$ , was taken to be the same as that for 2MH in MFI and their absolute values (see

Tab. 1) were tuned to match the zero-loading diffusivity value  $D_i(0) = 1.35 \times 10^{-12} \text{ m}^2 \text{ s}^{-1}$  obtained in the experiments.

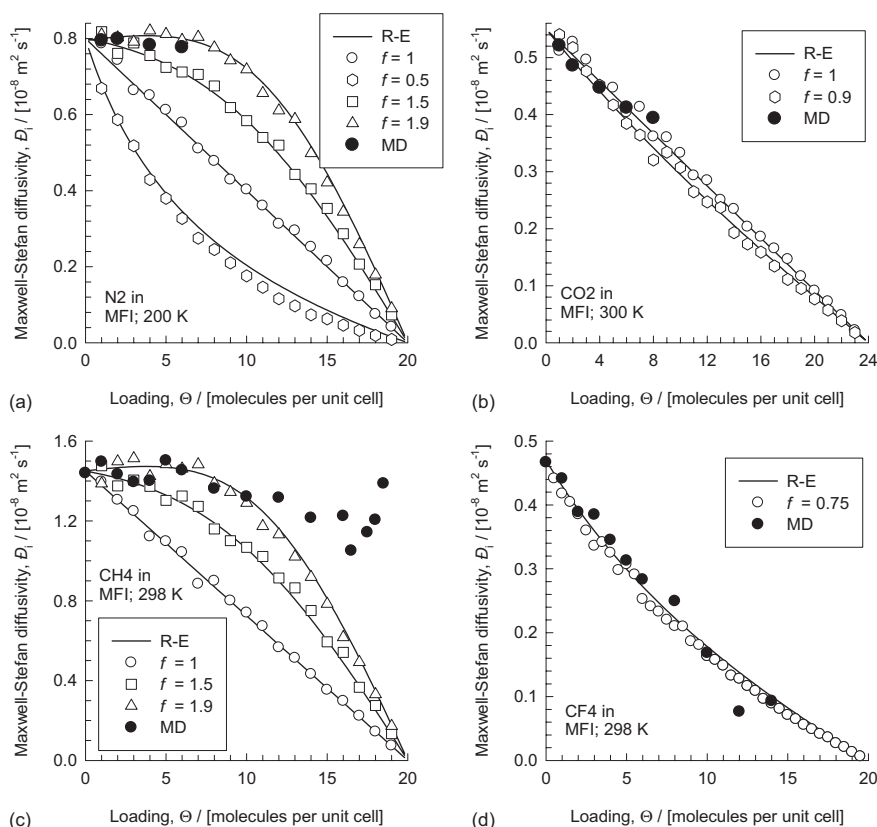
The filled black circles in Fig. 3d representing the experimental data on  $D_{i,self}$  correspond to the KMC simulations taking  $f = 1.5$ . The experimental data on the loading dependence can be rationalized if we take the energy barrier for diffusion of 3MP in MFI to be reduced due to increased loading. The Reed-Ehrlich model, taking  $z = 4$ , provides an accurate mean-field model for the KMC simulation results of  $D_i$  for all values of  $f$ . Furthermore Eqs. (5) and (6), with  $a_1 = 1.4$  and  $a_2 = 0.5$  provide an excellent description of the KMC simulations of the self-diffusivity.

In Fig. 4a we present the KMC simulation results of the M-S diffusivity  $D_i$  of  $\text{N}_2$  in MFI at  $T = 200 \text{ K}$ . The jump frequencies, specified in Tab. 1, were tuned to match the zero-loading diffusivity  $D_i(0) = 0.8 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$  obtained in the MD simulations reported by Papodopoulos *et al.* [12]. The number of sorption sites per unit cell was taken to be 20 on the basis of the sorption isotherm data [12]. The loading dependence of  $D_i$  for  $\text{N}_2$  obtained from their MD data can be rationalized by KMC simulations taking  $f = 1.9$ . In order to perform the calculations with the Reed-Ehrlich model we also need to specify the coordination number  $z$ . For  $\text{N}_2$  we have a total of 20 sorption sites, of which 4 are located at the intersections. Each of the intersection sites sees a maximum

of 4 neighbors. The other 16 sites are located in the channel interiors, 8 each in the straight and zig-zag channels. A channel site sees only 2 neighboring molecules. Therefore the average coordination number  $z$  can be calculated as  $z = (16 \times 2 + 4 \times 4) / 20 = 2.4$ . The continuous lines in Fig. 4a are the Reed-Ehrlich model calculations taking  $z = 2.4$  and the corresponding  $f$  value chosen for the KMC simulations. The agreement with the KMC simulations is excellent.

In Fig. 4b we present the KMC simulation results of the M-S diffusivity  $D_i$  of  $\text{CO}_2$ , in MFI at  $T = 300 \text{ K}$ . The jump frequencies, specified in Tab. 1, were tuned to match the zero-loading diffusivity  $D_i(0) = 0.55 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$  obtained in the MD simulations reported by Papodopoulos *et al.* [12]. The number of sorption sites per unit cell was taken to be 24 on the basis of the sorption isotherm data [12]. The loading dependence of  $D_i$  for  $\text{CO}_2$  obtained from their MD data can be rationalized by KMC simulations taking  $f = 0.9$ . In order to perform the calculations with the Reed-Ehrlich model we also need to specify the coordination number  $z$ . For  $\text{CO}_2$  we have a total of 24 sorption sites, of which 4 are located at the intersections. Each of the intersection sites sees a maximum of 4 neighbors. The other 20 sites are located in the channel interiors; 8 in the straight channels and 12 within the zig-zag channels as shown in Fig. 2. As before the average coordination number can be calculated from  $z = (20 \times 2 + 4 \times 4) / 20 = 2.33$ . The continuous lines in Fig. 4b are the Reed-Ehrlich model calculations taking  $z = 2.33$  and the corresponding  $f$  value chosen for the KMC simulations. The agreement with the KMC simulations is again excellent.

Let us now consider the KMC simulations for diffusion of  $\text{CH}_4$  in MFI at  $298 \text{ K}$ . From the sorption isotherm data we take the number of sorption sites per unit cell to be 20 [8]. The jump frequency along the straight channels is taken as  $\nu_{str} = 4 \times 10^{11} \text{ s}^{-1}$ , for transport along the zig-zag channels we take  $\nu_{zz} = 2 \times 10^{11} \text{ s}^{-1}$ . These jump frequency values are chosen so as to match the zero-loading diffusivity  $D_i(0) = 1.45 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ , in the x, y and z directions, obtained from MD [8]. The number of sorption sites within the MFI lattice is taken to be 20 per unit cell (4 at intersections, 8 within straight channels, and 12 within the zig-zag channels). Simulations have been carried out taking the pair interaction factors  $f = 1, 1.5, \text{ and } 1.9$ . The KMC results for  $D_i$  are presented as open symbols in Fig. 4c. Up to a loading  $\Theta_i$  of 10, the MD results (filled black circles) can be matched



**Figure 4.** KMC simulations (open symbols) of M-S diffusivity  $D_i$  in MFI of (a)  $\text{N}_2$  at  $200 \text{ K}$ , (b)  $\text{CO}_2$  at  $300 \text{ K}$ , (c)  $\text{CH}_4$  at  $298 \text{ K}$ , and (d)  $\text{CF}_4$  at  $298 \text{ K}$ . Also shown using continuous solid lines are the calculations of  $D_i$  with the Reed-Ehrlich model, Eq. (3). The filled black circles represent the MD simulation results [8, 12].

with the KMC simulations taking  $f = 1.9$ . The continuous lines in Fig. 4c represent calculations of the Reed-Ehrlich model taking  $z = 2.4$ , a value calculated above for  $N_2$  with a total of 20 sorption sites.

Let us now consider the KMC simulations for diffusion of  $CF_4$  in MFI at 298 K. We take the number of sorption sites per unit cell to be 20, despite the fact that the sorption isotherm indicates a maximum of 16 [8]; the reason for choosing 20 sites is that we later wish to simulate diffusion of a mixture of  $CH_4$  and  $CF_4$ . In KMC simulations of mixtures, to be described later, it is not possible to cope with different number of sorption sites for the constituent species. The jump frequency along the straight channels is taken as  $\nu_{str} = 1.2 \times 10^{11} \text{ s}^{-1}$ ; for transport along the zig-zag channels we take  $\nu_{zz} = 7 \times 10^{10} \text{ s}^{-1}$ . These jump frequency values are chosen so as to match the zero-loading diffusivity  $D_i(0) = 0.47 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ , in the x, y and z directions, obtained from MD [8]. The MD results (filled black circles) can be matched with the KMC simulations taking  $f = 0.75$ ; see Fig. 4d. The continuous lines in Fig. 4d represent calculations of the Reed-Ehrlich model taking  $z = 2.4$ ; the agreement with KMC simulations is excellent.

For diffusion of  $CH_4$  in the one-dimensional channels of MTW (ZSM-12), the M-S diffusivity  $D_i$  shows a slight inflection at a loading of 7 molecules per unit cell [9]; see Fig. 5a. This inflection is caused by the isotherm inflection, shown in Fig. 5b. The continuous lines in Fig. 5b represent the dual-site Langmuir isotherm

$$\Theta(p) \equiv \Theta_A + \Theta_B = \frac{\Theta_{sat,A} b_A p}{1 + b_A p} + \frac{\Theta_{sat,B} b_B p}{1 + b_B p} \quad (7)$$

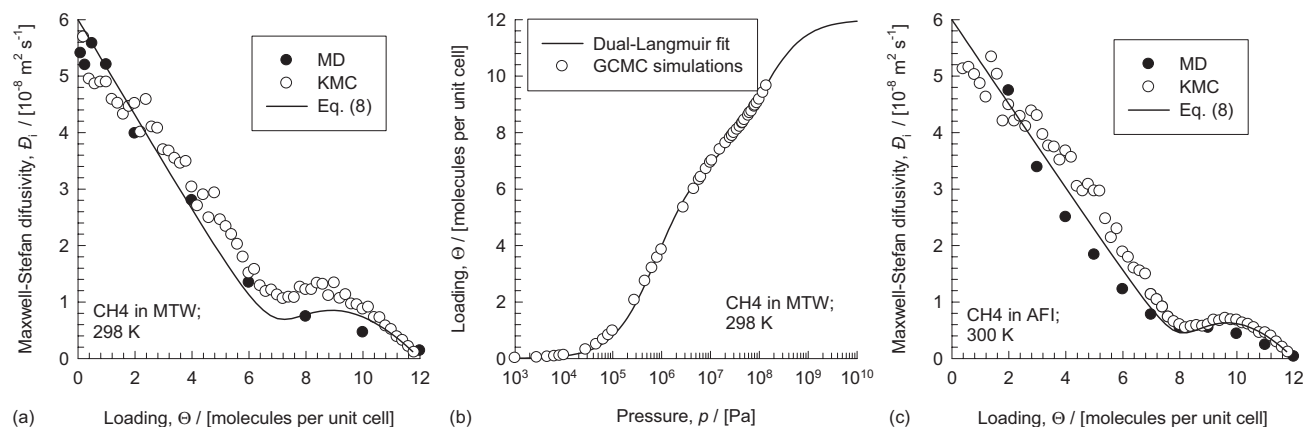
where the saturation capacities  $\Theta_{sat}$  of sites A (strong) and B (weak) are 7 and 5 molecules per unit cell, respectively. The corresponding Langmuir parameters are  $b_A = 1.29 \times 10^{-6} \text{ Pa}^{-1}$  and  $b_B = 8.65 \times 10^{-9} \text{ Pa}^{-1}$ . The KMC simulations of  $D_i$  (for topological details see [21]) taking the jump frequencies

$\nu = 4 \times 10^{11} \text{ s}^{-1}$  are shown by the open symbols in Fig. 5a. To capture the inflection behavior of  $D_i$  we take the jump frequencies *away* from the weak sites B to be a factor  $b_A/b_B = 150$  higher than the jump frequencies *towards* the weak sites, a KMC strategy explained in our earlier publication [10]. With this strategy the inflection in  $D_i$  occurs at  $\Theta = 7$ , faithful to the MD simulations. The continuous line in Fig. 5a is the extension of the scenario described by Eq. (2), derived earlier [10]:

$$D = D(0) [x_A (1 - \Theta_A / \Theta_{sat,A}) + x_B (1 - \Theta_B / \Theta_{sat,B})] \quad (8)$$

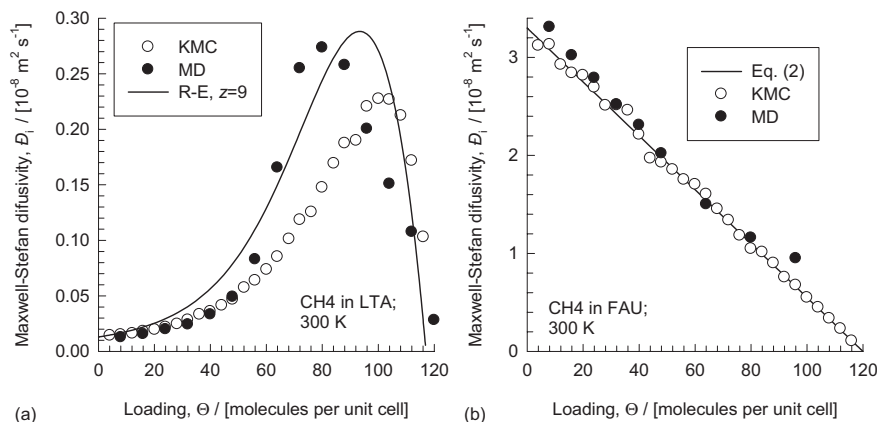
where we take  $D(0) = 6 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ . A similar loading dependence of  $D_i$  is found for diffusion of  $CH_4$  in the one dimensional channels of AFI as evidenced by the MD simulations results [25] (filled black circles), which show an inflection at a loading of 8 molecules per unit cell; see Fig. 5c. The KMC simulations of  $D_i$  (for topological details see [21]) taking the jump frequencies  $\nu = 1.4 \times 10^{12} \text{ s}^{-1}$  are shown by the open symbols in Fig. 5c. To capture the inflection behavior of  $D_i$  we take the jump frequencies *away* from the weak sites B, four in number, to be a factor  $b_A/b_B = 270$  higher than the jump frequencies *towards* the weak sites where the values  $b_A = 5.51 \times 10^{-4} \text{ Pa}^{-1}$  and  $b_B = 2.05 \times 10^{-6} \text{ Pa}^{-1}$  are obtained from GCMC simulations. The continuous line in Fig. 5c is the scenario described by Eq. (8), where we take  $D(0) = 6 \times 10^{-8} \text{ m}^2 \text{ s}^{-1}$ .

For diffusion of  $CH_4$  in the cage structure of LTA (zeolite 5A), with *narrow* windows separating the cages, the M-S diffusivity  $D_i$  shows a strong increase with loading, due to the reduction of the diffusion of the energy barrier [26]; see the black filled circles in Fig. 6a. KMC simulations (for topological details see [21]) can be made to reproduce the loading dependence by taking the rate limiting *intercage* jump frequency to be  $\nu = 2 \times 10^9 \text{ s}^{-1}$  and the factor  $f = 2$ ; see the open symbols in Fig. 6a. The continuous solid lines represent the



**Figure 5.** (a) KMC simulations (open symbols) of M-S diffusivity  $D_i$  of  $CH_4$  in MTW at 298 K. Also shown using continuous solid lines are the calculations of  $D_i$  using Eq. (8). The filled black circles represent the MD simulation results [9]. (b) GCMC simulations [9] of the isotherm for  $CH_4$  in MTW at 298 K. The continuous lines represent the dual-site Langmuir fits using Eq. (7). (c) KMC simulations (open symbols) of M-S diffusivity  $D_i$  of  $CH_4$  in AFI at 300 K. Also shown using continuous solid lines are the calculations of  $D_i$  using Eq. (8). The filled black circles represent the MD simulation results [25].





**Figure 6.** (a) KMC simulations (open symbols) of M-S diffusivity  $\mathcal{D}_i$  of  $\text{CH}_4$  in LTA at 300 K. The filled black symbols represent the MD simulation results of Beerdse *et al.* [26]. Also shown using continuous solid lines are the calculations of  $\mathcal{D}_i$  with the Reed-Ehrlich model, Eq. (3), taking  $z = 9$ . (b) KMC simulations (open symbols) of M-S diffusivity  $\mathcal{D}_i$  of  $\text{CH}_4$  in FAU at 300 K. The filled black symbols represent the MD simulation results of Chempath *et al.* [27]. Also shown using continuous solid lines are the calculations of  $\mathcal{D}_i$  using Eq. (2).

Reed-Ehrlich model taking  $f = 2$  and  $z = 9$ . The value of the coordination number is dictated by the fact that each of the 15 sorption sites within a cage is connected with 9 neighboring sites for jumps.

A different loading dependence is found for diffusion of  $\text{CH}_4$  in the cage structure of FAU wherein the cages are separated by *large* windows; witness the MD simulation results of Chempath *et al.* [27], shown by the black filled circles in Fig. 6b. The KMC simulations (for topological details see [21]) can be made to reproduce the loading dependence by taking the rate jump frequency to be  $\nu = 9.5 \times 10^{11} \text{ s}^{-1}$ ; see the open symbols in Fig. 6b. The continuous solid lines in Fig. 6b represent calculations according to Eq. (2).

### 3 Mixture Diffusion

Skoulidas *et al.* [28] have reported MD simulations for a mixture of  $\text{CH}_4$  and  $\text{CF}_4$  in MFI zeolite at 298 K. From their data the following modified matrix of  $[A]$  of Onsager diffusivities, defined by

$$(N) = -\rho \Theta_{sat} [A] \frac{1}{RT} (\nabla \mu) \quad (9)$$

can be determined wherein we take  $\Theta_{sat} = 20$  molecules per unit cell; these simulation results are shown by the filled black symbols in Figs. 7a, b, c, and d for mixtures with 75 %, 50 %, 30 % and 20 % methane, respectively.

The corresponding KMC simulation results, with the inputs as obtained from tuning *pure* component MD data and listed in Tab. 1 are shown by the open symbols. We note that the KMC simulations are able to match the MD data very well for all four mixture compositions for the whole range of

loadings. The continuous lines in Fig. 7 represent the predictions of the M-S mixture model [8, 19, 27]

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{\mathcal{D}_1} + \frac{\theta_2}{\mathcal{D}_{12}} & -\frac{\theta_1}{\mathcal{D}_{12}} \\ -\frac{\theta_2}{\mathcal{D}_{21}} & \frac{1}{\mathcal{D}_2} + \frac{\theta_1}{\mathcal{D}_{21}} \end{bmatrix}^{-1} \begin{bmatrix} \theta_1 & 0 \\ 0 & \theta_2 \end{bmatrix} \quad (10)$$

The Reed-Ehrlich formula is used for the estimation of the pure component  $\mathcal{D}_i$  and the binary exchange coefficients  $\mathcal{D}_{ij}$  are estimated from the pure component self-exchange coefficients  $\mathcal{D}_{ii}$  and  $\mathcal{D}_{jj}$  by the interpolation formula developed earlier [8, 19, 27]

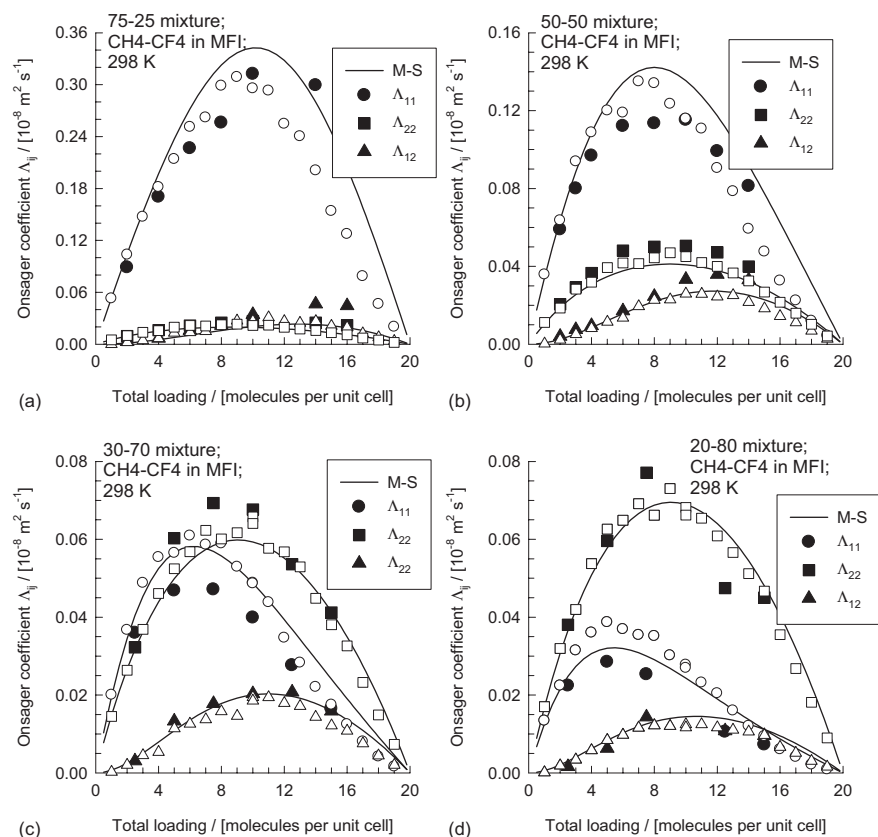
$$\Theta_{j,sat} \mathcal{D}_{ij} = [\Theta_{j,sat} \mathcal{D}_{ii}]^{\Theta_i / (\Theta_i + \Theta_j)}$$

$$[\Theta_{i,sat} \mathcal{D}_{jj}]^{\Theta_j / (\Theta_i + \Theta_j)} = \Theta_{i,sat} \mathcal{D}_{ji} \quad (11)$$

with the additional simplification here that the saturation capacities of the individual species are taken to be the same ( $= 20$ ). The M-S mixture model also does a very good job of predicting the mixture diffusion characteristics.

### 4 Conclusions

In this study we have shown that the KMC simulations can be tuned to reproduce the loading dependences of the M-S diffusivity  $\mathcal{D}_i$  observed by MD simulations or experiments in various zeolite topologies. By allowing the jump frequencies,  $\nu$ , of molecules to be influenced altered by a factor  $f = \exp(\frac{\delta E}{RT})$  for each neighboring molecule a variety of loading dependences can be realized. Furthermore, KMC



**Figure 7.** MD simulations (filled symbols) of Onsager matrix  $[A]$  for 75–25, 50–50, 30–70, and 20–80 mixtures of  $\text{CH}_4/\text{CF}_4$  in MFI at 298 K compared with KMC simulations (open symbols). The continuous solid lines represent calculations from the M-S mixture model, with pure component parameter values are specified in Tab. 1.

simulations allow mixture diffusion to be probed without additional parameter tuning.

The KMC simulations also validate the applicability of the quasi-chemical theory of Reed and Ehrlich [22] for describing the loading dependence of  $\bar{D}_i$ .

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### Symbols used

$a_i$	[-]	constants defined in Eq. (6)
$b$	$[\text{Pa}^{-1}]$	Langmuir adsorption constants defined in Eq. (7)
$D_{i,\text{self}}$	$[\text{m}^2 \text{s}^{-1}]$	self-diffusivity
$\bar{D}_i$	$[\text{m}^2 \text{s}^{-1}]$	Maxwell-Stefan diffusivity of species $i$ in zeolite

$\bar{D}_i(0)$	$[\text{m}^2 \text{s}^{-1}]$
$\bar{D}_{ii}$	$[\text{m}^2 \text{s}^{-1}]$
$\bar{D}_{ij}$	$[\text{m}^2 \text{s}^{-1}]$
$E$	$[\text{J mol}^{-1}]$
$f$	[-]
$n_i$	[-]
$n_{\text{sites}}$	[-]

$N_i$	$[\text{molecules m}^{-2} \text{s}^{-1}]$
$p$	$[\text{Pa}]$
$R$	$[\text{J mol}^{-1} \text{K}^{-1}]$
$t$	$[\text{s}]$
$T$	$[\text{K}]$
$x_A$	[-]
$z$	[-]

zero-loading Maxwell-Stefan diffusivity of species  $i$  in zeolite  
self-exchange diffusivity  
binary exchange diffusivity  
activation energy for diffusion  
parameter defined by Eq. (4)  
number of molecules of species  $i$   
number of sites in KMC simulation  
molecular flux of species  $i$   
pressure  
gas constant  
time  
absolute temperature  
fraction of total molecules occupying site A  
coordination number of lattice

### Greek symbols

$\beta$	[-]	parameter defined by Eq. (4)
$\varepsilon$	[-]	parameter defined by Eq. (4)
$\nu$	$[\text{s}^{-1}]$	jump frequency

$\theta$	[-]	fractional occupancy of component $i$
$\Theta$	[molecules uc <sup>-1</sup> ]	molecular loading
$\Theta_{sat}$	[molecules uc <sup>-1</sup> ]	saturation loading
$[A]$	[m <sup>2</sup> s <sup>-1</sup> ]	matrix of Onsager coefficients
$\mu_i$	[J mol <sup>-1</sup> ]	molar chemical potential
$\rho$	[uc m <sup>-3</sup> ]	zeolite density

### Subscripts

A	strong site
B	weak site
i, j	components in mixture
sat	referring to saturation conditions
st	referring to straight channels
zz	referring to zig-zag channels
1	component 1 in binary mixture
2	component 2 in binary mixture

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